Chapter 3: Probability Topics

3.1: Terminology

- **Outcome** - the result of an experiment
- **Event** - any combination of outcomes
- **Sample Space** - the set of all possible outcomes of an experiment
- **Probability** - a long-term relative frequency of an outcome whose value is between zero and one, inclusive
  - A probability of zero means that the event is **impossible**
  - A probability of one means that the event is **certain to occur**
- **Equally Likely** - when each outcome of an experiment occurs with equal probability
  - In a fair, six-sided die, it is equally likely to roll any of the numbers one through six.
  - In a fair coin, it is equally likely to get heads and tails.
  - However, it is not equally likely to get one heads and two heads when flipping a coin twice.
- The **complement** of event, A, denoted A', is the set of outcomes not in A and equal to $1 - P(A)$.
- The **conditional probability** of A given B, denoted $P(A|B)$, is the probability that event A will occur given that event B has already occurred.

3.2: Independent and Mutually Exclusive Events

- Two events are **independent** if the knowledge that one occurred does not affect the chance the other occurs.

- Mathematically, this means that $P(A|B) = P(A)$.

- Sampling **with replacement** means that members of the population are replaced after being selected and that each member can be chosen more than once (this is **independent** sampling because the result of the second pick **is not** affected by the first pick).

- Sampling **without replacement** means that members of the population are not replaced after being selected and that each member cannot be chosen more than once (this is **dependent** sampling because the result of the second pick **is** affected by the first pick).

- **Mutually exclusive** events cannot occur at the same time, meaning that they share no common outcomes and $P(A \text{ and } B) = 0$.

Example 1. Events A, B, and C are all equally likely and mutually exclusive.

(a) Find $P(A)$.

(b) Find $P(A')$.

(c) Find $P(A \text{ and } B)$.

3.3: Two Basic Rules of Probability

- The **multiplication rule** - the probability of two events both happening, $P(A \text{ and } B)$, is equal to $P(A)P(B|A)$.
  - If events A and B are independent, this simplifies to $P(A)P(B)$.
- The **addition rule** - the probability of either one of two events happening, $P(A \text{ or } B)$, is equal to $P(A) + P(B) - P(A \text{ and } B)$.
  - If events A and B are mutually exclusive, this simplifies to $P(A) + P(B)$.

Example 2. A spinner has 4 equal sectors colored yellow, blue, green, and red.

(a) What is the probability the spinner lands on yellow?

(b) What is the probability the spinner lands on red or blue?

(c) What is the probability the spinner does NOT land on green?