BENFORD'S LAW AND HUMANLY GENERATED PRICES IN AUCTION HOUSES AND BUYOUT SYSTEMS OF VIRTUAL WORLDS

A THESIS

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BY

MEGAN ENDRESS A.A, B.S.

DENTON, TEXAS

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TEXAS WOMAN'S UNIVERSITY
DENTON, TEXAS

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To the Dean of the Graduate School:

I am submitting herewith a thesis written by Megan Endress entitled "Benford's Law and Humanly Generated Prices in Auction Houses and Buyout Systems of Virtual Worlds." I have examined this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science with a major in Mathematics

Dr. David Marshall, Major Professor

We have read this thesis and recommend its acceptance:

Brent Lilley

Ann Wheeler

Don E. Edwards

Department Chair

Accepted:

Dean of the Graduate School

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ABSTRACT

MEGAN ENDRESS

BENFORD'S LAW AND HUMANLY GENERATED PRICES IN AUCTION HOUSES AND BUYOUT SYSTEMS OF VIRTUAL WORLDS

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The purpose of this study was to analyze the buyout, or "buy now," prices in auction houses of virtual environments, such as World of Warcraft and Guild Wars 2. Human players interact with an auction house user interface in order to buy or sell in-game items, purchasable with in-game currency. Players wishing to sell items can post their items on the auction house for set lengths of time, as well as set a starting bid amount and/or an amount in which other players can instantly buy the item. Since the establishment of Benford's Law, it has been supported that data generated by humans typically does not follow Benford's Law, proving to be a beneficial tool in detecting fraudulent accounting data. However, this study shows that the leading significant digits of these buyout prices in virtual environments created by humans follow Benford's Law by utilizing Kuiper's goodness of fit V_n test, a modified Kolmogorov-Smirnov test.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER I

INTRODUCTION

A teacher wrote the numbers one through nine on a sheet of paper, cut them separately, placed them in a bowl, held the bowl up high, and asked a student to pull out one number. She asked the class, "What is the probability that the student pulled a two?" Intuitively, the class replied in unison, "One-ninth." Students learn in elementary school about probability and learn in elementary statistics about the uniform distribution, such as this case. Imagine now the surface areas of every river in the United States, but only think of the first digit of each area. Does each of those nine digits also have an equal one-ninth probability of showing? To one's surprise, the leading digits of these areas would not follow a uniform distribution, but a special logarithmic distribution known as Benford's Law.

In fact, stock market prices, city populations, the numbers appearing in *Readers Digest* and street addresses from people mentioned in *American Men of Science* are a few among some of the data collected by Frank Benford when he established "The Law of Anomalous Numbers", or Benford's Law as it is more commonly known as today. He credited this phenomenon to anomalous numbers that appear to have no known relation. By his definition, numbers that follow some common relation deviate the most from this distribution law. Benford noted that the distribution of the first occurring digits of his gathered data, followed closely to the logarithmic distribution found by previous founder, Simon Newcomb, the first person documented to have mentioned the existence of the phenomenon. More importantly, what Newcomb and Benford found is an intriguing principal that has fascinated statisticians, economists, physicists and others over the last century. What makes this law so interesting is how it breaks intuition that the leading significant digits of multi-digit numbers should have an equal, probable chance of occurring, $P(1) = P(2) = P(3) = \cdots = P(9) = 0.11$.

The phenomenon of Benford's Law is typically seen in naturally occurring data sets, such as the ones listed above. In these cases, the leading significant digits occur in decreasing frequency, from one to nine.

Many scholars have shown the applications and usefullness of Benford's Law. Consequently, Mark Nigrini showed in *The Use of Benford's Law as an Aid in Anayltical Prodecures* how the law can be used to help auditors detect accounting fraud. Nigrini wrote that since original numbers are naturally occuring and should conform to Benford's Law, intintionally forged numbers would be noticably different in distribution and henceforth detectable in an audit. On the contrary, David Giles considered the case of eBay closing prices of professional football games and found that these prices, while humanly created and bidded on, showed to follow Benford's Law.

Similar to Giles studies, this thesis focused on auction houses of virtual gaming worlds to determine if the buyout, or "buy now," prices also followed Benford's Law. We considered two large, popularly played massively multi-player online roleplay games (MMORPG), Blizzard Entertainment's World of Warcraft and Arena Net's Guild Wars 2. In these MMORPGs, human players interact within a virtual environment through a player's character, controling movement and appearance, earning experience points for completing tasks and objectives in order to advance in levels, which dictates what the player can do and where they can go. While the development of their character is the primary goal, players also socialize with characters of other human players and non-playing characters (npc). A common feature of World of Warcraft and Guild Wars 2 is the auction house. Here, players interact with an auction house user interface (UI) to buy or sell in-game items, purchasable with in-game currency. This boosts the already existing virtual economy within the gaming evnironment (Castronova 173). Players wishing to sell items can post their items on the auction house for set lengths of time, as well as set a starting bid amount or even an amount where other players can instantly buy them, similar to eBay's "buy now" option.

The purpose of this study was to consider the prices of items being sold at the "buy now" option and the leading significant digits to determine if these listed prices followed Benford's Law. We were not concerned if the auctions were actually bought or not, nor were we considering any negotiated prices or bidding; we were only concerned with the price at which they were listed for immediate purchase. This was accomplished by utilizing Kuiper's goodness of fit V_n test to determine if the distribution of the sample prices differs from Benford's distribution. There is a variety of statistical tests that could be administered here to determine if a data set conforms to Benford's Law. However, we discuss the limitations of other such tests in chapter III.

CHAPTER II

REVIEW OF LITERATURE

Before the introduction of handheld electronic calculators in the 1970s, logarithmic tables simplified large, arduous calculations. Logarithmic tables reduced multiplication and division problems into a matter of addition and subtraction through logarithms with base ten. Numbers were written in exponential form $b^x = y$, and converted to logarithmic form, $log_b(y) = x$, where x is the value found in the logarithmic table and y could be found using the anti-logarithm table. Someone using these techniques also had to identify the characteristic, or location of the decimal point, and the mantissa, the fractional part of the logarithm that is found in the table. For example, to find the logarithm of 150, through some manipulation we get $log_{10}150 = log_{10}(10^2 \cdot 1.5) = log_{10}(10^2) + log_{10}(1.5) \approx 2 + 0.176091$, where the characteristic is 2 and the mantissa is 0.176091. These documents were heavily used because this was the widely used method for over four hundred years to calculate surveying, navigation, chemistry, engineering and much more (Johnston "Slide Rule").

In 1881 astronomer Simon Newcomb wrote a short article for the *American Journal of Mathematics* where he noted that the first pages in logarithmic tables wore out faster and appeared to have higher use on the first pages versus the last pages which showed little wear and tear. He concluded that the first digit was one more often and the frequency decreased up to nine and identified this property as "the Law of Frequency." Since naturally occurring numbers are considered ratios of quantities, Newcomb said that in order to consider the probability of choosing a number with the first significant digit d, one must pick two numbers and determine the probability of their ratio (Newcomb 39-40).

He proposed that the probability distribution of the mantissa, the fractional parts of the logarithm, will approach a uniform distribution around a circle as the number of d increases. "The law of probability of the occurrence of numbers is such that all mantissa of their logarithms are equally probable." That is to say, the digits one through nine could be arranged around a circle where each leading digit *d* has an equal one-ninth chance of occurring. However, as we pass nine and place more sequential numbers around the circle (i.e., ten, eleven, and twelve) we have more digits with a leading significant digit of one. This would raise our previous probability of digit one occurring and lower the previous probabilities of the other eight digits, until we reach twenty and the same would occur for raising the probability for the digit two and lowering the others, and so on until we begin to approach a circular uniform distribution (Newcomb 39-40).

We then took the first 100,000 numbers, from 1 to 100,000, and considered the accumulative probabilities of the leading digit *d*, as seen in the following line plots. For instance, with just 1, the probability of showing a number with the leading digit of one is $\frac{1}{1}$. When we have 1 and 2, the probability of showing a number with the leading digit of one is $\frac{1}{2}$. This line plot shows in each case that as the total sample size n increases, the probabilities of digit d occurring as the leading significant digit converges to 11%. However as n gets larger and larger, the average for each digit's probability will correspond to the logarithmic frequency established by Newcomb (see figure 1).

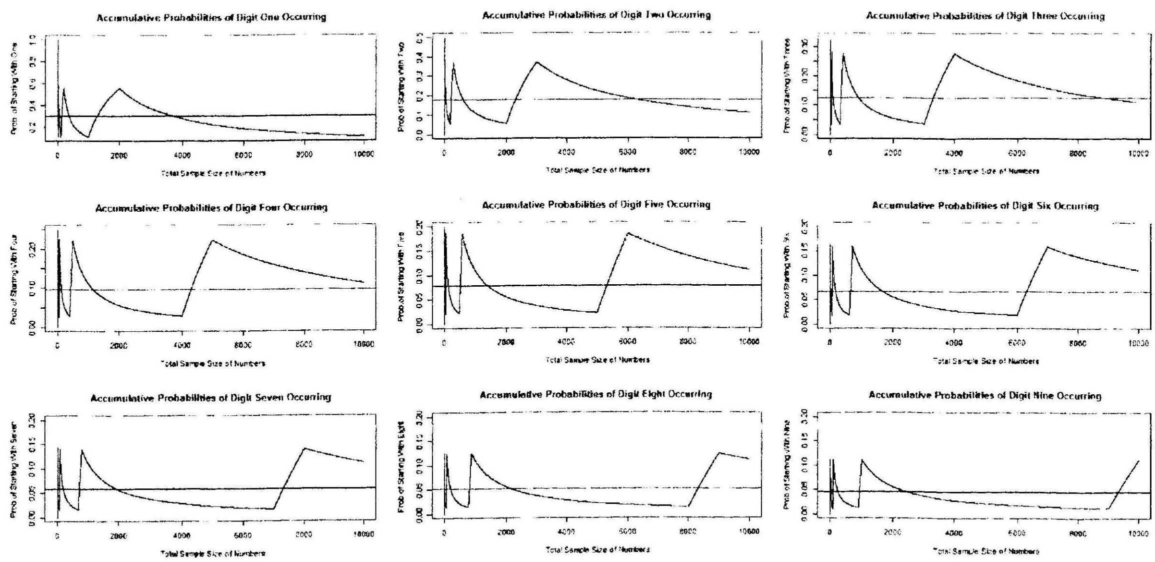

Figure 1. Accumulative Probabilities of Digit *d*

Together on a linear scale, the probabilities follow a logarithmic distribution (see equation 2.1 and table 1, where *d* is the leading significant digit of interest and *d'* is the second number).

$$P(d) = \log(d+1) - \frac{\log(d)}{\log(10)} - \log(1) = \log\left(\frac{d+1}{d}\right) \tag{2.1}$$

Table 1

Probabilities of Occurrence for the First Two Significant Digits of a Natural Number

d	P(*d*)	P(*d'*)
0	. . .	0.1197
1	0.301	0.1139
2	0.176	0.1088
3	0.125	0.1043
4	0.097	0.1003
5	0.079	0.0967
6	0.067	0.0934
7	0.058	0.0904
8	0.051	0.0876
9	0.046	0.085

Source: Newcomb, Simon. "Note on the Frequency of Use of the Different Digits in Natural Numbers." *American Journal of Mathematics*. 4.1 (1881): 40. Web. 7 Mar. 2014.

In 1938 physicist Frank Benford authenticated Newcomb's claim by assessing naturally occurring sets of data and tabulated data, such as city populations, river surface areas, atomic weights, and molecular weights, to show if their distribution was similar to a logarithmic distribution (see table 2). The twenty different fields of collected data varied from arbitrary numbers of information to proper mathematical calculations (Benford 553). It is unclear whether Benford had prior knowledge of Newcomb's work on the subject; however, Benford noticeably worked more in depth with empirical data to establish its validity and provide further information and examples.

Table 2

Percentage of Times the Natural Numbers 1 to 9 are Used as First Digits in Numbers, as Determined by 20, 229 Observations

Group	Title	First Digit 1	2	3	4	5	6	7	8	9	Count
A	Rivers, Area	31.0	16.4	10.7	11.3	7.2	8.6	5.5	4.2	5.1	335
B	Population	33.9	20.4	14.2	8.1	7.2	6.2	4.1	3.7	2.2	3259
C	Constants	41.3	14.4	4.8	8.6	10.6	5.8	1.0	2.9	10.6	104
D	Newspapers	30.0	18.0	12.0	10.0	8.0	6.0	6.0	5.0	5.0	100
E	Spec. Heat	24.0	18.4	16.2	14.6	10.6	4.1	3.2	4.8	4.1	1389
F	Pressure	29.6	18.3	12.8	9.8	8.3	6.4	5.7	4.4	4.7	703
G	H.P. Lost	30.0	18.4	11.9	10.8	8.1	7.0	5.1	5.1	3.6	690
H	Mol. Wgt.	26.7	25.2	15.4	10.8	6.7	5.1	4.1	2.8	3.2	1800
I	Drainage	27.1	23.9	13.8	12.6	8.2	5.0	5.0	2.5	1.9	159
J	Atomic Wgt.	47.2	18.7	5.5	4.4	6.6	4.4	3.3	4.4	5.5	91
K	$n^{-1}, \sqrt{n} \ldots$	25.7	20.3	9.7	6.8	6.6	6.8	7.2	8.0	8.9	5000
L	Design	26.8	14.8	14.3	7.5	8.3	8.4	7.0	7.3	5.6	560
M	*Digest*	33.4	18.5	12.4	7.5	7.1	6.5	5.5	4.9	4.2	308
N	Cost Data	32.4	18.8	10.1	10.1	9.8	5.5	4.7	5.5	3.1	741
O	X-Ray Volts	27.9	17.5	14.4	9.0	8.1	7.4	5.1	5.8	4.8	707
P	Am. League	32.7	17.6	12.6	9.8	7.4	6.4	4.9	5.6	3.0	1458
Q	Black Body	31.0	17.3	14.1	8.7	6.6	7.0	5.2	4.7	5.4	1165
R	Addresses	28.9	19.2	12.6	8.8	8.5	6.4	5.6	5.0	5.0	342
S	$n^1, n^2 \ldots n!$	25.3	16.0	12.0	10.0	8.5	8.8	6.8	7.1	5.5	900
T	Death Rate	27.0	18.6	15.7	9.4	6.7	6.5	7.2	4.8	4.1	418
	Average	30.6	18.5	12.4	9.4	8.0	6.4	5.1	4.9	4.7	1011
	Probable Error	± 0.8	± 0.4	± 0.4	± 0.3	± 0.2	± 0.2	± 0.2	± 0.2	± 0.3	--

Source: Benford, Frank. "The Law of Anomalous Numbers." *Proceedings of the American Philosophical Society*. 78.4 (1938): 553. Web. 7 Mar. 2014.

Through his findings, he was able to come to a general overview that the more inherently random the numbers appeared to be, the closer they would follow Newcomb's principal and that the more tabulated data veered the most. He came to rename the principal "the Law of Anomalous Numbers" because the groups that he found to follow this principal appeared anomalous, having no apparent relation until examined in whole (Benford 551). However, over time Benford's Law of Anomalous Numbers would become more popularly known as "Benford's Law."

The frequencies from his data he found closely mimicked Newcomb's logarithmic distribution, but he also noticed that this distribution law only applied to large numbers of three or more digits. Benford went further to state that while sets with numbers comprised of three or more digits created a logarithmic series, sets with numbers comprised of single or double digits only created a geometric series (Benford 563). He compared his observed and computed frequencies (see table 3) and noticed his observed frequencies were comparable to the expected frequencies.

Table 3

Observed and Computed Frequencies

Natural Number	Number Interval	Observed Frequency	Logarithm Interval	Observed - Computed	Prob. Error of Mean
1	1 to 2	0.306	0.301	+ 0.005	± 0.008
2	2 to 3	0.185	0.176	+ 0.009	± 0.004
3	3 to 4	0.124	0.125	- 0.001	± 0.004
4	4 to 5	0.094	0.097	- 0.003	± 0.003
5	5 to 6	0.08	0.079	+ 0.001	± 0.002
6	6 to 7	0.064	0.067	- 0.003	± 0.002
7	7 to 8	0.051	0.058	- 0.007	± 0.002
8	8 to 9	0.049	0.051	- 0.002	± 0.002
9	9 to 10	0.047	0.046	+ 0.001	± 0.003

Source: Benford, Frank. "The Law of Anomalous Numbers." *Proceedings of the American Philosophical Society*. 78.4 (1938): 554. Web. 7 Mar. 2014.

Many scholars have aimed to show instances where Benford's Law applies. An important property of the law came through Theodore Hill when he proved Benford's Law is scale invariant, $P(kx) = f(k)P(x)$, where k is some constant (Hill "A Statistical Derivation" 354-363). Also noticeably, Mark Nigrini wrote multiple articles about the applications of Benford's Law and how it can be used in an analytical process to detect accounting fraud. Ideally, genuine data, such as stock market prices and even accounting data can be expected to follow Benford's Law because they contain anomalous numbers. Therefore, anyone attempting to influence the data for personal gain would not be considering such distributions and would create any ideal number that seems to best fit their current situation of fraud. Such fraudulent input or alteration to data is easy to detect by looking at the posterior distribution to see if it deviated from Benford's distribution (Nigrini and Mittermaier 52-67).

Additionally, Hsü, Kubovy and Hill conducted similar experiments asking individuals to create unique four, five and six digit numbers, respectfully, and discovered no Benford relationship. In all three, as well as with Nigrini's work, results indicate that humans do not make well for random number generators nor are they decent at creating data conforming to Benford's Law (Hsü 57-67; Kubovy 359-364; Hill "Random Number-Guessing" 967-971).

While others strove to show that numbers randomly created by humans did not follow Benford's Law, Bruce Burns worked to show the contrary. He believed that human thought followed a Benford distribution but attributed the lack of relationship to manner in which the random information was requested or given. Burns suggests that when asked to create numbers based on something meaningful, as opposed to entirely arbitrary, people are more likely to follow Benford's law. He showed this was possible through two studies with psychology students, asking for numerical answers for nine questions. In both cases, findings were close to Benford's Law (Burns "Sensitivity to Statistical Regularities").

Likewise, David Giles considered the closing prices of winning bids of eBay auctions for professional football games to see if human influence on bid prices would conform or defy the principal. For analytical purposes, he excluded college football game tickets, all Dutch auctions, and auctions won through the "buy-it-now" option. Giles utilized Kuiper's test because he claimed his data had distributional circularity, making traditional testing for Benford's Law less desirable. This test was also ideal to him because it is invariant for cyclic transformations, while the Kolmogorov-Smirnov test or Chi-Square Goodness of Fit test are not (Giles "Benford's Law" 157-167)

Kuiper's test has a crucial property that distinguishes it from the Kolmogorov-Smirnov. It is rather fitting for observations that lie around a circle because "If the observations are points on a circle, the value of V_N does not depend on the choice of origin for measuring x (Stephens 309)." Therefore, Kuiper's test is a suitable test when cyclic data is the topic at hand.

For example, consider the probability that an individual is born in a certain month. The months are initially thought of as linearly distributed, where January is strictly at the beginning of the year and December is strictly at the end of the year (see figure 2).

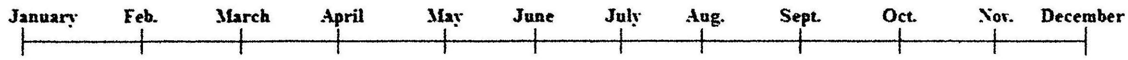

Figure 2. Linear Model of Calendar Months

Yet, the months can also be thought of as being organized around a circle, where January and December are next to each other (see figure 3), as December comes right before January (Giles, "Testing for a Santa Clause" 422).

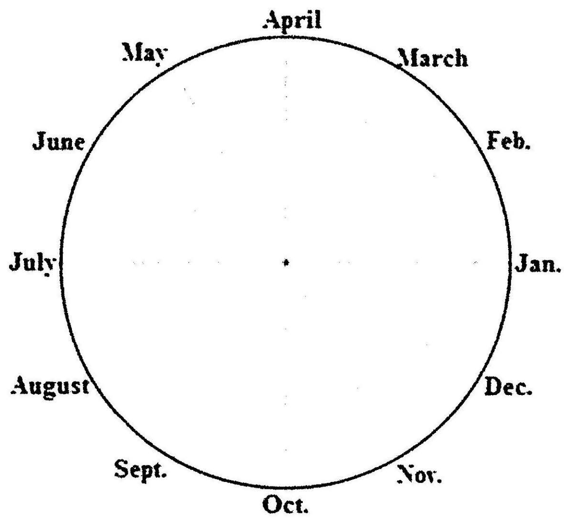

Figure 3. Calendar Months Organized Around A Circle

Similarly in Benford data, numbers are thought of in a linear pattern, ranging on the number scale of $-\infty < x < \infty$, where there is a distinguishable difference between one and nine (see figure 4).

Figure 4. Number Line

However, for money data, although digits one and nine are on opposite ends of the number line, their leading significant digits can be thought of as being organized around a circle just as Newcomb proposed. As far as Benford rules are concerned, prices \$0.10, \$1.00 and \$10.00 are relatively the same because they all have the same first digit of one.

For instance, when the prices \$0.90 and \$1.00 are considered, their first digits one and nine are not psychologically considered at opposite ends of the spectrum, rather two contingent discrete numbers where the first price might have undercut the second price by ten cents.

Equivalently, when the prices \$0.99 and \$1.00 are considered and their second digits are taken into consideration, a similar psychological occurrence is seen, where they are considered as two continuous discrete numbers where one undercut the other by a penny (see figure 5). Therefore, through this phenomenon one might expect to see a larger concentration of the digit nine than is forecasted by Benford's distribution.

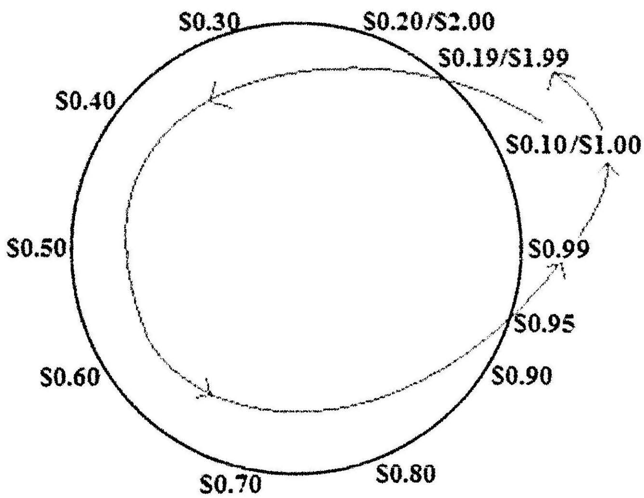

Figure 5. Prices Organized Around a Circle

Accordingly, Giles used Kuiper's test, a customized version of the Kolmogorov-Smirnov test, which is more suitable to meet the needs of such circular data, to determine if the eBay data conformed. Similar to Burns, Giles found that the winning eBay prices showed to conform to Benford's Law (Giles, "Benford's Law" 157-161).

CHAPTER III

RESEARCH METHOD

Because both MMORPG's have such a large player base, servers are needed to host the vast amounts of data transferred between each player and the virtual environment. Guild Wars 2 hosts more than 460,000 players (O'Brien). Similarly, World of Warcraft hosts over 7.7 million players in the US alone (Karmali). To help alleviate the large amounts of active players, multiple servers are utilized to host each game's virtual environment. Guild Wars 2 refers to their different servers as "Worlds" while World of Warcraft refers to theirs as "Realms." Worlds in Guild Wars 2 are all the same type, where players can freely and openly participate in Player versus Environment (PvE), interacting and engaging in combat against computer NPCs, as well as Player versus Player (PvP), interacting and engaging in combat against other human players and role-playing (RP) with other characters and NPCs through the game where they act out some role or idea in game through the persona of their character. Meanwhile, in World of Warcraft, there are four different combinations of the types of realms that players can choose to play on.

Data showing posted auction house prices for every currently posted item is made freely available through an Application Programming Interface, API, per the gaming company's website (i.e. ArenaNet and Blizzard). The files containing all the information are obtained in JavaScript Object Notation format, JSON. Because auction house information is not static, JSON files are continuously updated every hour from each server of each game. The JSON files for Guild Wars 2 are available in a single file, encompassing data from all worlds, while JSON files for World of Warcraft has to be pulled individually for each named realm. Since there are different player compositions among server types and between the two games, for the intent of collecting a more accurate portrayal of auction house prices, data was pulled from several servers of each five server types from World of Warcraft and the one available from Guild Wars 2. The servers from World of Warcraft that were selected were chosen from high and low player populations so that they would have a combined player population total that was similar to other total player populations from each group (e.g. PvE, PvP, etc...). However, it is important to note that the total player population of PvP-RP servers is substantially lower than the other four realm types due to the limited number of servers of this type. When data was pulled, there were only six PvP-RP servers; consequently, data was pulled from all six to be as consistent as possible. At the time of pulling, data was pulled from the following servers and had the approximate player populations, given in table 4. For this study, data for World of Warcraft was only pulled from US servers because the player populations on Oceanic and Brazilian servers were low ("US Realm Pop"). There was no selection option available for gathering data from Guild Wars 2 as only one pull file was available.

Table 4

World of Warcraft Realms and Player Population as of February and March, 2014

Realm Name	Player Population	Realm Name	Player Population
PvE RP		PvP RP	
Cenarion Circle	70,660	Emerald Dream	136,659
Earthen Ring	94,639	Lightninghoof	42,031
Feathermoon	83,274	Maelstrom	42,451
Moon Guard	151,607	Ravenholdt	38,234
Steamwheedle Cartel	41,076	The Venture Co	30,729
Wyrmrest Accord	134,473	Twisting Nether	43,048
Total	575,729	Total	333,152
PvE		PvP	
Area 52	197,913	Blackrock	133,346
Proudmoore	170,258	Sargeras	184,711
Stormrage	213,485	Tichondrius	269,062
Total	581,656	Total	587,119

Source: n.p. US Realm Pop. Realm Pop, n.d. Web. 1 Mar 2014. <http://wow.realmpop.com/us.html>.

Data was pulled every two hours on February 9th, 2014 and March 1st, 2014 between noon, Central Standard Time, and midnight for each of the nineteen JSON pulls between World of Warcraft and Guild

Wars 2. These two days were chosen partly because the only other known similar study, by Giles, was taken over about a week's span and obtained 1,161 "successful" eBay auctions (Giles, "Benford's Law" 159). Due to continuously updating data and the large number of player inputs to auction house posts through these virtual environments, a week's span was not necessary in this case study; thereupon two days felt sufficient. They were also chosen because the weekends are generally the busiest days for player participation because of work and school during the weekdays. In addition, the weekends are the busiest for social interaction and group assisted combat for PvE. Therefore the weekend presents optimal days to attempt to sell goods on the auction house.

Once the data was pulled, each JSON files were converted to a Comma Separated Values format (CSV) for easier extraction of the initial significant digits, using a combination of a converter service at www.json-csv.com and the text import wizard in excel. For each file through excel, a new column was created for the first significant digit using the "=left(text, [num_chars])" command in order to extract only the first significant digit. In total, there were 14,479,583 auction house prices posted for sale over the entirety of the eighteen chosen World of Warcraft servers and the single Guild Wars 2 data file between the two days of pulling.

In the study of eBay auction prices by Giles, Kuiper's test was shown to be ideal with this particular type of data due to its circularity. He noted that Chi-Square and Kolmogorov-Smirnov Goodness of Fit Tests have been historically used in assessing Benford's Law. These tests, however, did not account well for the circularity that was present in his data set, nor ours (Giles, "Benford's Law" 157-161).

Kuiper's V_n test is a modified Kolmogorov-Smirnov test. What makes this test so unique to our data is that it is scale invariant (251-252). We use it to test if the significant digits in our set follow the same distribution as Benford's Law. Let $F_0(x) = log\left(\frac{1+a}{a}\right)$.

$$H_0: empirical\ CDF\ for\ sample\ size\ N, F_N(x), follows\ the\ population\ distribution, F_0(x)$$

$$H_a: empirical\ CDF\ for\ sample\ size\ N, F_N(x), does\ not\ follow\ the\ population\ distribution, F_0(x)$$

Kuiper's test statistic is the following:

$$V_N = D_N^+ + D_N^- \tag{3.1}$$

The two summed parts are the one-sided Kolmogorov-Smirnov statistics as follows:

$$D_N^+ = max[F_N(x) - F_0(x)] \tag{3.2}$$

$$D_N^- = max[F_0(x) - F_N(x)] \tag{3.3}$$

Critical values are calculated for numbers larger than one hundred and the critical values at the 10%, 5% and 1% significance levels are 1.620, 1.747 and 2.001 respectfully (Stephens 310).

Furthermore, an additional property through Kuiper's test that is not met by a traditional Kolmogorov-Smirnov test is that the null distribution $F_0(x)$ is invariant to the hypothesized distribution, $F_n(x)$(Kuiper 252).

Initial descriptive statistics are assessed through excel by calculating the frequency in which each number appeared in the 14 million numbers, as well as a bar graph, to see if it appears approximately Benford, which are addressed in Chapter IV.

To be thorough, we ran the data through several hypothesis tests to compare and evaluate findings at each level.

CHAPTER IV

RESULTS

The visual representation of the combined two days showed that the distribution of the auction house first significant digits looked extremely similar to the Benford's Law distribution, (see figure 1). In fact, the data follow a Benford distribution rather nicely, and it appears that the major hypothesis of this study received prima facie support.

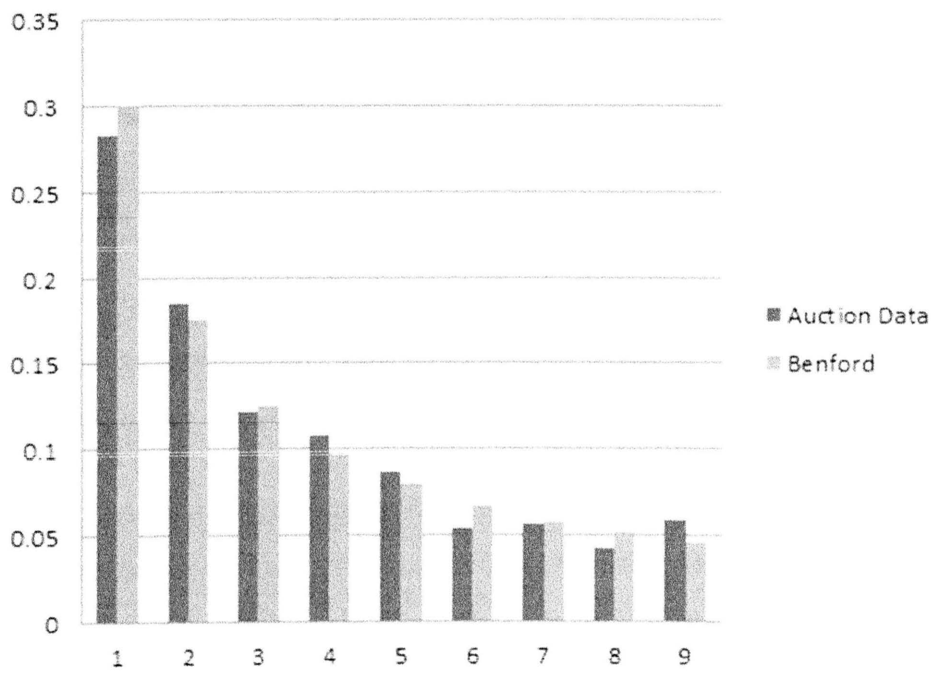

Figure 6. Distribution Comparison of Virtual Environment Auction Houses and Benford's Law

It is clear from first visual that our auction house data is not a uniform distribution, so testing for uniformity was not necessary. It is also apparently not from a normal distribution, so testing for normality was also unnecessary.

Data was pulled every two hours on February 9th, 2014 and March 1st, 2014 between noon, Central Standard Time, and midnight for each of the nineteen JSON pulls between World of Warcraft and Guild Wars 2. These two days were chosen partly because the only other known similar study, by Giles, was taken over about a week's span and obtained 1,161 "successful" eBay auctions (Giles, "Benford's Law" 159). Due to continuously updating data and the large number of player inputs to auction house posts through these virtual environments, a week's span was not necessary in this case study; thereupon two days felt sufficient. They were also chosen because the weekends are generally the busiest days for player participation because of work and school during the weekdays. In addition, the weekends are the busiest for social interaction and group assisted combat for PvE. Therefore the weekend presents optimal days to attempt to sell goods on the auction house.

Once the data was pulled, each JSON files were converted to a Comma Separated Values format (CSV) for easier extraction of the initial significant digits, using a combination of a converter service at www.json-csv.com and the text import wizard in excel. For each file through excel, a new column was created for the first significant digit using the "=left(text, [num_chars])" command in order to extract only the first significant digit. In total, there were 14,479,583 auction house prices posted for sale over the entirety of the eighteen chosen World of Warcraft servers and the single Guild Wars 2 data file between the two days of pulling.

In the study of eBay auction prices by Giles, Kuiper's test was shown to be ideal with this particular type of data due to its circularity. He noted that Chi-Square and Kolmogorov-Smirnov Goodness of Fit Tests have been historically used in assessing Benford's Law. These tests, however, did not account well for the circularity that was present in his data set, nor ours (Giles, "Benford's Law" 157-161).

Kuiper's V_n test is a modified Kolmogorov-Smirnov test. What makes this test so unique to our data is that it is scale invariant (251-252). We use it to test if the significant digits in our set follow the same distribution as Benford's Law. Let $F_0(x) = log\left(\frac{1+a}{a}\right)$.

H_0: $empirical\ CDF\ for\ sample\ size\ N, F_N(x), follows\ the\ population\ distribution, F_0(x)$

H_a: $empirical\ CDF\ for\ sample\ size\ N, F_N(x), does\ not\ follow\ the\ population\ distribution, F_0(x)$

Kuiper's test statistic is the following:

$$V_N = D_N^+ + D_N^- \quad (3.1)$$

The two summed parts are the one-sided Kolmogorov-Smirnov statistics as follows:

$$D_N^+ = max[F_N(x) - F_0(x)] \quad (3.2)$$

$$D_N^- = max[F_0(x) - F_N(x)] \quad (3.3)$$

Critical values are calculated for numbers larger than one hundred and the critical values at the 10%, 5% and 1% significance levels are 1.620, 1.747 and 2.001 respectfully (Stephens 310).

Furthermore, an additional property through Kuiper's test that is not met by a traditional Kolmogorov-Smirnov test is that the null distribution $F_0(x)$ is invariant to the hypothesized distribution, $F_n(x)$(Kuiper 252).

Initial descriptive statistics are assessed through excel by calculating the frequency in which each number appeared in the 14 million numbers, as well as a bar graph, to see if it appears approximately Benford, which are addressed in Chapter IV.

To be thorough, we ran the data through several hypothesis tests to compare and evaluate findings at each level.

CHAPTER IV

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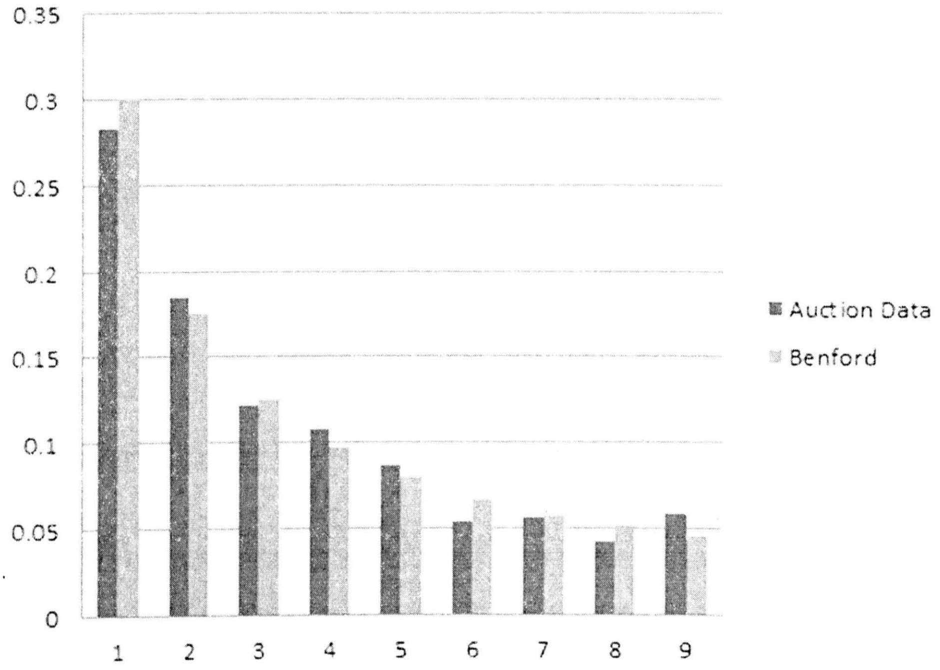

Figure 6. Distribution Comparison of Virtual Environment Auction Houses and Benford's Law

It is clear from first visual that our auction house data is not a uniform distribution, so testing for uniformity was not necessary. It is also apparently not from a normal distribution, so testing for normality was also unnecessary.

Chi-Square Goodness of Fit is used to determine how close the empirical proportions are to the theoretical proportions that we are interested in. The Chi-Squared distribution with k degrees of freedom is given by

$$f(\chi^2) = \frac{1}{2^k \Gamma(k/2)} e^{-\chi^2/2} (\chi^2)^{(k/2)-1}. \tag{4.1}$$

It is tested using the form, when using observed and empirical proportions

$$\chi^2 = 14{,}437{,}976 \sum \frac{(observed - expected)^2}{expected}. \tag{4.2}$$

A random variable χ^2 has (n – 1) degrees of freedom with at least two values of n; it is an appropriate test to use with nominal variables, and assumes that we are using a large sample size and that there is independence between all observations. When the test statistic returns a large value, the empirical and theoretical values are considered not close and therefore the model is not a good fit to the data.

We first ran the chi-square goodness of fit test against the following null and alternative hypotheses.

$$H_0: The\ proportions\ from\ the\ auction\ houses\ follow\ the\ proportions\ from\ Benford's$$

$$H_a: The\ proportions\ from\ the\ auction\ houses\ do\ not\ follow\ the\ proportions\ from\ Benford's$$

When we accessed the data, when α = 0.05, $\chi^2(8)$ returns a value of 162,225.15. With such an extremely large value, we very quickly rejected the null hypothesis for this case that the proportions of the auction house data were similar to the proportions of Benford.

The Kolmogorov-Smirnov test is often referred to as the "K-S test" for short. It is used to establish the equality of one or two distributions, and is a non-parametric test and assumes nothing about the distribution of the data. The two-sample K-S test is more informative than the one-sample K-S test in that it is sensitive to location and shape difference of the observed data's cumulative distribution function for each observed distribution.

After the Chi-Square Goodness of Fit test, we ran the K-S test on the data against the following null and alternative hypotheses:

$$H_0\text{: } The\ auction\ house\ data\ follows\ Benford's\ distribution$$

$$H_a\text{: } The\ auction\ house\ data\ does\ not\ follow\ Benford's\ distribution$$

The K-S test concentrates on the largest deviation between the theoretical and empirical differences, given by the following, where $F(x)$ is the theoretical cumulative distribution function and $F_N(x)$ is the empirical cumulative distribution function:

$$D = \max[F(x) - F_N(x)] \qquad (4.3)$$

Our sample size was extremely large, at 14,437,976; with sample sizes over fifty the critical value is calculate by $\frac{1.36}{\sqrt{N}}$, where N is the sample size. When $\alpha = 0.05$, this gave us a critical value of 0.000358.

The K-S test returned a max value of 0.018311. This value, being greater than our critical value, required that we reject the null hypothesis that the auction house data follows Benford's distribution. Looking at the figure 5 above, we can see that the frequency of digit nine might influence the K-S test values. As previously mentioned, the K-S test does not account for circularity, and we know that digits one and nine can be thought of being arranged around a circle and would be right next to each other even though they are at opposite ends of the range.

The Kuiper test is a modified version of the Kolmogorov-Smirnov test, as it also tests an empirical distribution of observed data against an expected theoretical distribution to see how much they differ. What sets Kuiper's test apart from the K-S test though is that the Kuiper test is not dependent on the location of the origin. This test proves highly useful when observations lie on a circle. Its test statistic is similar to the K-S test, as it uses the maximum difference between the theoretical and empirical cumulative distribution functions, but also uses the maximum difference between the empirical and theoretical cumulative distribution functions. The test statistic is given by

$$V_N = D_N^+ + D_N^-, \tag{4.4}$$

where the two summed parts of the one-sided Kolmogorov-Smirnov statistics are

$$D_N^+ = max[F_N(x) - F_0(x)] \tag{4.5}$$

$$D_N^- = max[F_0(x) - F_N(x)] \tag{4.6}$$

When we ran Kuiper's goodness of fit test, we produced results that made most sense given that the data and theoretic distributions appear so similar. We ran the Kuiper test on the data against the following null and alternative hypotheses:

$$H_0: The\ auction\ house\ data\ follows\ Benford's\ distribution$$

$$H_a: The\ auction\ house\ data\ does\ not\ follow\ Benford's\ distribution$$

Calculating the Kuiper test statistic gave us 0.0313; at the $\alpha = 0.05$ significance level, the critical value as 1.747. Since our test statistic was less than the critical value, we failed to reject the null hypothesis that the distribution of the auction house data follows closely to the distribution of Benford's Law, which is visibly supported by the graphs shown in figure 5 above.

The results of each hypothesis testing are given in table 5.

Table 5

Comparisons of Statistical Testing

Test	n	Critical Value, $\alpha = 0.05$	Test Statistic
Chi-Square Goodness of Fit	14,437,976	15.507	162,225.15
Kolmogorov-Smirnov	14,437,976	0.000358	0.018311
Kuiper	14,437,976	1.747	0.0313

CHAPTER V

CONCLUSION

Benford described anomalous numbers as being "numbers that individually are without relationship, and when considered in large group are in good agreement with a distribution law" (Benford 551). These auction house prices can be viewed as anomalous numbers because individually, they appear to have no relationship to one another. When viewed collectively, they present a trend that follows Benford's Law, opposite of what most would typically expect to see in mass data generated by humans.

Our efforts to verify Benford's Law in auction house data in virtual environments of World of Warcraft and Guild Wars 2 were not shown successful using the typical tests, chi-square goodness of fit or Kolmogorov-Smirnov. This could be due to the extremely large sample size used, as sufficiently large samples may reject null hypotheses for any departure in the data, no matter how slight.

However, utilizing Kuiper's test to account for circularity in the data showed that Benford's Law cannot be rejected. The presence of Benford's Law is evident when visually accessing the distribution of the auction house data compared to the distribution of Benford's Law. This is similar to the one analyzed by Giles, who showed in his work how the winning bids of eBay prices conformed to Benford's Law.

The higher amount of the digit nine in our data did pose a potential concern if it would affect the statistical test results. This could have played a role in the rejection of the null hypotheses in the chi-square test and the Kolmogorov-Smirnov test; however, it was not enough to negatively affect the Kuiper test. Additionally, we can attribute the higher amount of the digit nine to several factors: undercutting (a seller posting an item on the auction house at a lower price than what is already posted), outside program add-ons or "mods" (e.g. Auctioneer or Auctionator), or embellishing a highly sought after item for mass profit. Identifying if an auction price is posted by use of an add-on or completely through human element presents a challenge, as it is difficult to identify exactly how many players are using these add-ons. Even so, even if a player is actively using an add-on that assists with posting auction prices, the player has complete control over if they choose to use the add-on in game, or when they choose to use it.

Implications for future research might include better psychological understanding of how video gamers think according to Benford's Law or why the presence of nine is higher in these cases. Additionally, research might extend to marketing analysis in these types of cases, to follow these trends in auctions for suggestions of price rigging. Extensive data collection could even possibly present a new law of conformity in association with auction house data that is similar to Benford's Law.

We have shown here that auction house prices presented in World of Warcraft and Guild Wars 2 appear to conform to Benford's Law, but perhaps further studies could be done to include other popular, mainstream MMORPGs to determine if their player based auction house data also conforms to Benford's Law.

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APPENDIX A

IRB Approval Letter

TWU
TEXAS WOMAN'S UNIVERSITY
DENTON DALLAS HOUSTON

Institutional Review Board
Office of Research and Sponsored Programs
P.O. Box 425619, Denton TX 76204-5619
940-898-3378 FAX 940-898-4416
e-mail: IRB@twu.edu

December 13, 2013

Ms. Megan Endress

Dear Ms. Endress:

Re: Benford's Law and Humanly Generated Prices in Auction Houses and Buyout Systems of Virtual Worlds (Protocol #: 17561)

The above referenced study has been reviewed by the TWU Institutional Review Board (IRB) and was determined to be exempt from further review.

If applicable, agency approval letters must be submitted to the IRB upon receipt PRIOR to any data collection at that agency. Because a signed consent form is not required for exempt studies, the filing of signatures of participants with the TWU IRB is not necessary.

Any modifications to this study must be submitted for review to the IRB using the Modification Request Form. Additionally, the IRB must be notified immediately of any unanticipated incidents. If you have any questions, please contact the TWU IRB.

Sincerely,

Rhonda R Buckley

Dr. Rhonda Buckley, Chair
Institutional Review Board - Denton

cc. Dr. Don Edwards, Department of Mathematics & Computer Science
Dr. David Marshall, Department of Mathematics & Computer Science
Graduate School