THE EFFECTS OF READABILITY OF MATHEMATICAL MATERIALS ON ACHIEVEMENT IN AN EIGHTH GRADE MATHEMATICS CLASS

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TABLE OF CONTENTS

Chapter		Page
	ACKNOWLEDGMENTS	iv
	LIST OF TABLES	vii
	LIST OF FIGURES	viii
I.	INTRODUCTION	1
	Statement of the Problem	8
	Purpose of the Study	9
	Null Hypothesis	10
II.	REVIEW OF THE LITERATURE	11
	Historical Development of Readability Formulas	11
	Application of Formulas to Mathematics Materials	14
	Readability Formulas Developed for Mathematics	16
III.	METHODS AND PROCEDURES	19
	Subjects	19
	Materials	20
	Procedures	24
	Analysis	28
	Limitations	29
	Definition of Terms	30
IV.	ANALYSIS	32
V.	SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS	39
	Conclusions	40
	Recommendations	41
	APPENDICES	45
	A. Procedures for Using Kane's Formula II	46

Chapter Page

B. Application of Dale-Chall Readability Formula 53
C. Estimating the Precision of Kane's Formula II* 57
D. Information Sheet 62
E. Data Employed in the Study 64
F. Pretest-Posttest 69
G. Procedure for Using the Dale List of 3000 Easy Words* 72
H. List of Mathematics Words Familiar to 7th-8th Grade Students* 98
I. List of Mathematics Words Not Familiar to 7th-8th Grade Students * 101
J. List of Mathematics Symbols Known by 7th-8th Grade Students * 108
K. Mathematics Lessons 110
L. Predicted Cloze Scores for Lessons A, B, and C 141

REFERENCES 147

LIST OF TABLES

Table		Page
1.	Comparison of the Range of Pretest Scores and Posttest Scores	34
2.	Analysis of Pretest Scores	35
3.	Comparison of Means and Standard Deviations of Pretest Scores and Posttest Scores	36
4.	Analysis of Variance on Pretest Scores	36
5.	Analysis of Variance on Posttest Scores	37
6.	Analysis of Variance on Pretest/Posttest Difference	38

LIST OF FIGURES

Figure Page

1. Procedures Flowchart 25

CHAPTER I

INTRODUCTION

Mathematics is made up of systems of related ideas or concepts. It is a way of thinking. Evolving from the systems of ideas we call mathematics, man gradually developed a variety of overt behaviors such as counting, measuring, and computing. Although these procedures prove useful in tackling many day-to-day problems, the essence of mathematics is not merely a set of observable behaviors to be trained into a child. Such behaviors, useful as they are, are only the visible signs of the fundamentals underlying them--the system of ideas. Moreover, these overt procedures are only useful to those who have acquired the concepts underlying them.

In recent years, many educational researchers have focused attention on children and their acquisition of mathematical knowledge. Although differences of opinion exist among these researchers as to how children learn mathematics, and to what extent they ought to learn mathematics, most agree that the mathematical development of children correlates highly with their ability to read mathematical materials (Earle, 1976). Further, the more meaningful the approach taken in learning mathematics the more important

reading ability becomes (Aaron, 1965).

The relationship between a learner's mathematical development and his ability to read mathematics is such that it is necessary for successful mathematics teachers also to be teachers of reading of mathematics materials.

The middle school mathematics curriculum has experienced a phenomenal change in recent years (Bitter, 1978). Bitter's research found that calculators, computers, and research on learning had the most influence on mathematics education. The most outstanding of these changes are the increased use of machines to replace drill and practice and the increased emphasis on problem solving. As these changes have developed, the middle schools have incorporated more individualized approaches to instruction.

Teachers are generally urged to include variety in their individualized teaching, but Roueche (1972) specifically cautions that "Self-instructional math units may lose their effectiveness if they offer only traditional, symbolic sets of readings" (p. 67). Even when a variety of techniques are used with the self-instructional math units, most information is still secured through silent reading with no verbal instruction. This fact presents a special problem for the student who cannot read at the level assumed by the instructor. Even though individualized instruction offers many advantages for the non-traditional student, including

adequate time for self-pacing, an important variable in student achievement is being overlooked when the readability of materials is not considered.

Lorge (1944) defines readability as "the quality of a piece of reading matter that makes it readable and understandable" (p. 407). Dale and Chall (1949) define the concept more completely:

> Readability is the sum total (including interactions) of all those elements within a given piece of material that affect the success that a group of readers have with it. The success is the extent to which they understand it, read it at an optimum speed, and find it interesting. (p. 20)

Researchers have repeatedly shown a relatively high correlation between reading ability and mathematical ability (Balow, 1964). General and specific reading abilities have been found to be correlated positively with scores on reading and mathematics tests. These investigations, the majority of which have been based on children in the middle grades, yielded correlations between .40 and .86 on reading ability and mathematics achievement (Balow, 1964; Chase, 1960; Cleland & Tousaint, 1962; Cottrell, 1968; Erickson, 1958; Muscio, 1962; Pitts, 1952; Earp, 1971).

Ryan's (1978) research on history textbook materials, a study in a content area other than mathematics, reported the greatest factors in comprehension of reading to be intelligence and motivation.

A readability formula is a linear equation which predicts reading difficulty. Until recently readability formulas existed only for measuring the reading difficulty of ordinary English passages. However, studies have been reported in which several of these formulas were used to rate the readability of mathematics textbooks. The most popular of these formulas which have been applied to mathematics texts and problems are the Flesch formula (1943), the Dale-Chall formula (1949), and the "cloze" procedure (Taylor, 1953).

The Flesch formula yields a reading ease score based on sentence length and word length for samples of reading material. Sentence length is determined by finding the average number of words per sentence. Word length is based on the average number of one-syllable words per hundred words. A modification of Flesch's original formula computes reading ease by the formula, "Reading Ease = 1.599S - 1.01N - 31.517" in which S represents the average number of one-syllable words per one hundred word sample and N represents the average number of words per sentence.

Attempts have been made to modify the Flesch formula for use specifically with mathematics. One variation of the formula was developed by Faison (1951). In using the Flesch formula, he counted each digit of large numbers as a word. He also considered each practice exercise as a

sentence since it represented a complete unit. However, Flesch suggests that arithmetic portions be omitted when applying the formula to mathematics materials.

The Dale-Chall formula gives a readability score based on average sentence length and vocabulary familiarity. The formula is as follows:

$$R = .1579X + .0496Y + 3.6365$$

when

R = Reading-grade score of a pupil who could answer correctly 50% of the questions on a reading comprehension test covering the passage

X = Number of words not on the Dale list of 3000 familiar words

Y = Average sentence length

A sample is taken every ten pages. Because of the nature of mathematics textbooks with formulas and symbols, an appropriate selection may not occur every tenth page. In that case, the next page with the appropriate material would be considered.

The Dale-Chall formula has been used in several studies to measure the readability of mathematics textbooks, especially on the middle school and secondary level. The most recent of these studies are Selikson (1951) and Heddens and Smith (1964), Wiegand (1967), Smith (1969), and Kane (1970).

The cloze procedure is another method which can be used to assess reading difficulty for ordinary English text.

Cloze tests are constructed by deleting every fifth word from a passage and replacing them with blanks of uniform size. There are no deletions from the first or the last sentence of the passage with fifty deletions. In cloze tests, students fill in the blanks with words they think that the author had written. The average percent of agreement between the subject's responses and the author's original words is called the "mean cloze score" for the passage. Above fifty-six percent is independent level; forty-four to fifty-six percent is the instructional level; below forty-four percent is frustration level for scoring the cloze test. Therefore, the higher the score the easier the material is to read. Cloze tests have several advantages over comprehension tests as a measure of reading difficulty: the style of the author is strictly retained, inferences cannot be imposed by the test constructor, and all sections of the passages are sampled equally, including charts and diagrams (Grouws & Robinson, 1973).

Results from research studies (Bormuth, 1962, 1963, 1966; Fletcher, 1959; Friedman, 1964; Gallant, 1965; Jenkinson, 1957; Rankin, 1957; Taylor, 1953, 1954, 1957) indicate that cloze tests are reliable and valid measures of reading comprehensibility and difficulty for ordinary English passages. However, Kane (1967, 1969, 1975) pointed out differences between ordinary English and mathematical

English and that the cloze procedure could not be assumed to be valid for use with mathematical English. Hater (1969, 1975), one of Kane's associates, conducted a study which validated the cloze procedure as a measure of the comprehensibility and difficulty of mathematical English.

In 1970 Kane continued to question the validity of applying readability formulas to mathematics materials when these formulas had originally been developed specifically for ordinary English. In his article, "The Readability of Mathematics Textbooks Revisited," which appeared in the Mathematics Teacher in November, 1970, Kane discussed some of his concerns about the use of the present readability formulas. Kane's first major concern was that large sections of mathematical material had to be omitted from the rating because it was presented in symbols rather than in English. A second major concern was that in rating a selection there is no way to account for mathematical vocabulary that is usually present.

Kane and his associates Hater and Byrne established a practical equivalency of mean cloze scores and mean comprehension scores for mathematical materials. As a result of this study, the criterion measure used in the formula was the mean score on a 75 item cloze test.

In determining the predictor variables, approximately 110 variables were originally defined and studied. The

variables studied are grouped into five categories to give an idea of the range of variables considered: vocabulary variables; symbol variables; special characteristics of mathematics materials; special classes of words such as prepositions, connectives, or quantifiers; and syntactic variables. These variables were tested carefully and eventually reduced to four predictor variables which Kane presented in the following formula:

$$R = -0.15A + 0.10B - 0.42C - 0.17D + 35.52$$

when

R = Predicted mean cloze score

A = Words not on Dale list of 3000 words and also that are not on the 80% mathematics word list

B = Number of changes from a word token to a math token and vice-versa (A word token is usually a single word, while a math token may be a symbol that represents several words).

C = Number of different mathematics terms not on 80% Mathematics List plus number of different math symbols not on 90% Symbol List

D = Number of question marks

Detailed instructions on the use of the formula are given in Appendix A. The information on Kane's formula II is taken from his published research (1974).

Statement of the Problem

A relatively high correlation has already been shown to exist between general reading ability and mathematical

ability as measured by traditional readability formulas. However, few correlations are known to exist between readability of mathematics materials and achievement in mathematics. The problem, then, is represented by the question, "Would mathematics materials having the same content, but also providing the option of various reading levels, significantly influence the learning of that content, that is, mathematics achievement?" For example, a mathematical concept may not be too difficult for a student to grasp, but may be obscured by difficult terminology and writing style. The student might be able to readily understand the same concept when it is presented at a lower reading level.

If readability of mathematical materials can be shown to be a significant variable in achievement outcome, the consideration of this factor could have important implications for the future. If in individualized instruction, the selection of appropriate mathematical materials is critical it might be equally as important to select the appropriate level of content material. It makes little difference how elementary the concepts may be if a student is unable to read the materials with understanding that are given to him.

Purpose of the Study

The purpose of the study was to determine the degree of relationship of the readability level of mathematics

materials on achievement for middle school students enrolled in regular eighth grade mathematics. The study investigated the question, Will varying the readability level of selected mathematics materials used with regular students in a middle school significantly alter the posttest scores of a given lesson?" A null hypothesis was generated from this question.

Null Hypothesis

There will be no statistically significant difference in the amount of learning evidenced between groups who received instruction from mathematics materials on varying levels of readability.

CHAPTER II

REVIEW OF THE LITERATURE

A survey of the literature revealed that the readability of reading materials is not a new concept. Readability formulas began to emerge over fifty years ago. The purpose of this chapter was to present a review of the development of these readability formulas that are related to this study.

Historical Development of Readability Formulas

As early as 1920 investigators became interested in readability, Thorndike's first list of word frequencies in English writings appeared in 1921 when he published his Teacher Word Book. He followed this list with two others in 1932 and 1944.

Tools for predicting readability were not developed until the latter part of the decade. Vogel and Washburne in 1928 made one of the first attempts to determine readability of children's books. The variables considered in the formula were the number of sentences that occurred in 1000-word samples, the number of uncommon words, the number of prepositions occurring in each sample, and the number of simple sentences found in each sample. "Uncommon words" was defined as the words which did not appear on the Thorndike list.

In March of 1929, Lewerenz reported a study conducted by the Educational Research Division of the Los Angeles Public Schools to investigate the relationship between letters of the alphabet and readability. Webster's Elementary Dictionary was used as the standard. The results of the study were that words beginning with the letters "w," "h," and "b," occurred more frequently in easy reading material. In difficult reading material, the letters "i" and "e" occurred more often.

In 1930 Johnson offered another variable in determining readability--the number of polysyllabic words occurring in the material. He studied this variable experimentally and concluded from his results that a test written in monosyllabic words was strikingly less difficult than the polysyllabic word test.

In 1943 Flesch published his readability formula which attracted much attention. His study was concentrated mostly on materials which were written for adult readers. Flesch considered two variables in his readability formula which had not been included in previous formulas. As well as the standard variables of difficult words, sentence lengths, etc., he accounted for the number of morphemes and the number of abstract words contained in a given passage.

Shortly after Flesch published his first readability formula, Lorge introduced another formula for measuring the

difficulty of reading material. His formula called for counting words, sentences, and prepositional phrases.

In 1948 Flesch revised his original formula by providing a means to measure what he referred to as the "human interest" qualities of a piece of writing. The main factor which was considered in this measurement was the number of personal references used in a given piece of writing.

Dale and Chall criticized the Flesch formula because it was too time consuming to use. They set about to develop a more efficient formula which would be more suitable for use with children's books than the Flesch formula. In 1948 Dale and Chall published a new readability formula which is still a popular formula today. Its two predictor variables are vocabulary familiarity and average sentence length.

Another popular formula was presented by Spache in 1953. This formula was intended for use with primary level reading materials.

It was also in 1953 that Taylor presented the "cloze" procedure for measuring reading difficulty. Cloze tests are constructed by deleting certain words from the text and replacing them with blanks of uniform size. Students fill in the blanks with words they believe the author has written. The average percent of agreement between the subjects' responses and the author's original words is called the "mean cloze score" for the passage.

Application of Formulas to Mathematics Materials

Several studies have tested the readability of mathematics textbooks by applying standard readability formulas (Cramer & Dorsey, 1969; Faison, 1951; Horn, 1978; Johnson, 1957; Smith, 1969; Smith & Heddens, 1964; Tinker, 1954; Wiegand, 1967; Williams, 1979). Because of the occurrence of mathematical symbols, readability formulas had to be modified somewhat when applied to mathematics materials. Johnson (1957) used a modified version of the Flesch formula to measure the readability of mathematics textbooks. As a result of his study, Johnson concluded that, "In view of the variation in readability of different textbooks for a given grade, readability scores should be used in the selection of a text" (p. 285). This statement was in reference to mathematics textbooks.

In addition to the Flesch formula, the Dall-Chall and Spache formulas have been utilized to determine the readability of mathematics materials. Faison (1951) used the Flesch formula in appraising mathematics and other textbooks in grades 5 through 8. Selikson (1951) applied the Flesch formula and the Dale-Chall formula to sixth grade mathematics textbooks.

Two studies by Heddens and Smith (1964a, b) applied the Spache formula to primary mathematics textbooks and the

Dale-Chall formula to intermediate mathematics materials. One of the major conclusions was that consideration of the readability levels of the mathematics materials is of great importance since it does not seem reasonable that a student can make efficient use of the materials if he is unable to read them. Smith and Heddens also noticed that in the textbooks that they evaluated that not all portions were equally readable within each book. Also, for a given series there seemed to be no consistent increase in reading difficulty.

In a subsequent article Smith (1965) continued to defend the use of readability formulas in mathematics and reemphasized his belief in the importance of the consideration of readability of materials:

> The importance of the readability factor has long been acknowledged and now it assumes vital importance in consideration of mathematics text materials. Certainly, the discovery method requires inductive thinking by children which is beyond that traditionally required, and the area of methematics has complexities which historically have created difficulties for children. Only as reading difficulty in mathematics text materials is considered and adjusted will there be any lessening of the confounding of reading achievement with mathematics understanding and proficiency. (p. 364)

In 1969 Smith conducted a study in which he determined the readability level of mathematics textbooks for grades seven and eight. The textbooks examined were those submitted to the Texas State Textbook Committee in 1967. From these results Smith concluded that the readability level was generally high and the range within a given textbook was

great. There was no consistent increase in the level of readability throughout a given series, but rather a "distribution of very easy and very difficult material throughout the books" (p. 290). He also expressed the need for careful selection of a textbook so that reading does not become more of a problem than teaching or learning the content. Serra's (1954) research on amplifying and simplifying instructional materials found that simplification of vocabulary had less effect on comprehension than does simplification of language structure and the relative difficulty of the concept involved.

Readability Formulas Developed for Mathematics

As has been shown, a number of researchers have used standard readability formulas in an attempt to measure the readability of mathematical textbooks. In these studies, mathematical words were usually classified as difficult words and mathematical symbols were ignored in applying the formula. Kane (1970) criticized the use of standard readability formulas for measuring the readability of mathematics materials.

A few researchers have made progress toward adapting regular readability techniques to the language of mathematics. The cloze procedure has been used by researchers as a measure of the readability of ordinary English. Hater

(1969) validated the cloze procedure as a measure of reading comprehension on the language of mathematics at grade levels seven through ten. Byrne (1970) established a measure of the familiarity of mathematical terms and symbols to seventh and eighth grade students in the United States.

Kulm (1973) studied the readability of elementary algebra textbooks. He used multiple regression analysis with criterion measure of mean cloze scores to search for predictors of readability. He found that the percentage of mathematical symbols, the percentage of mathematical words, average sentence length, and the percentage of questions were the best four predictors accounting for approximately forty percent of the variation. From these results, Kulm has not devised or published a formula to be used in measuring the readability of algebra textbooks.

Before a final decision was made to use Kane's Formula II, the Dale-Chall's readability formula was used to investigate the readability of the materials used in the study. Details of this investigation are given in Appendix B.

Building on Hater's and Byrne's research, Kane, Hater and Byrne (1974) constructed a readability formula for the language of mathematics. This formula predicts the mean cloze scores on mathematical passages.

From these various approaches to measuring readability, Kane's Formula II was selected as the most appropriate for

use with this particular study. It is the only readability formula derived specifically for use with mathematics materials, and even though it is a recent addition to the list of readability formulas, its validity and reliability have been successfully established (Kane, Hater & Byrne, 1974). Estimating the precision of Kane's Formula II is given in detail in Appendix C. These facts about the formula tend to increase the credibility of this study.

CHAPTER III

METHODS AND PROCEDURES

Since this study was designed as a field experiment, the first phase involved selecting a readability formula, securing a population, and developing instructional materials and instruments to measure achievement.

Subjects

The subjects used in the investigation were all of the students enrolled in three regular mathematics classes at an urban middle school in the Southwest during the third six weeks of spring 1981. These students were assigned to a regular eighth grade class because their total math score on the Iowa Test of Basic Skills (ITBS) was between 6.4 and 9.5. Seventy-four students comprised three sections of eighth grade mathematics, a required class for all eighth grade students. Information form appears in Appendix D which contained demographic information.

Even though student characteristics such as age, sex, and ethnic origin were not used as selection criteria, this information was used in describing the group. Of the 74 students, 46 were females and 28 were males. In terms of

ethnic origin, there were 3 Mexican Americans (M), 39 Caucasians (A), and 32 Blacks (B). The students ranged in chronological age from 13 to 15 with a mean age of 13.8. This information is presented in Appendix E.

Materials

It was necessary to select a mathematical concept about which the lessons could be written. Ideally, the material should have been totally unfamiliar to the students, but most students had been exposed at some time to the majority of the concepts presented in eighth grade mathematics. Therefore in order to select totally unfamiliar material, it was necessary to reach beyond the scope of the subjects' normal mathematical experiences. Using an unfamiliar topic would have provided a new learning experience rather than a review of a previous learning experience. However, the teachers of the three classes felt that the lessons must definitely cover material presented in the course and preferred that these lessons include material presented during the last semester of class due to time constraints. With these restrictions imposed, the subject of "mathematical sets" was chosen for the lessons. This material for these lessons appears in Appendix F.

The intent was to choose one mathematics topic to be written at three different levels of reading difficulty.

The material presented on each level was essentially the same except for the level of readability. The information pertaining to sets in Extending Mathematics (Dean, Kane, McMeen & Oesterle, 1963) was divided into three consecutive lessons. This material was left in its original form and became the lessons representing the readability level of medium difficulty (level B) and was used by the control group (B).

The other two levels of difficulty were achieved by manipulating the variables of Kane's Formula II to either increase or decrease the mean cloze score for the passages. Kane's Formula II and the predictor variables are given below:

$$R = -0.15A + 0.10B - 0.42C - 0.17D + 35.52$$

when

R = Predicted readability (means cloze score)

A = Words not on Dale List of 3000 Words (Appendix G) and also not on 80% Mathematics List (Appendix H and I)

B = Number of changes from a word token to a math token and vice-versa

C = Number of different mathematics terms not on 80% Math List and not on 90% Symbols List (Appendix J)

D = Number of questions marks

Since a higher mean cloze score represents an easier level of reading difficulty or readability, the predicted

readability score must be increased to make the passages easier to read and decreased to make them more difficult to read.

To increase the predicted readability score, the variables were manipulated in the following manner:

Since variable A, words not on Dale List of 3000 Words and also not on 80% Mathematics Word List, has a negative correlation with the mean cloze score, it was decreased which will increase the total score. If a word did not appear on the Dale List of 3000 Words or was not on the 80% Mathematics List, a word from one of these lists was substituted if at all possible. Of course, it was impossible to eliminate every word that did not appear on these lists because of content.

Variable B, the number of changes from a word token to a math token and vice-versa, had a positive relationship with the mean cloze score and needed to be increased. This was accomplished by breaking up sequences of math tokens with some explanations in words and vice-versa.

Variable C was the number of different mathematics terms not on 80% Math List plus the number of different mathematics symbols not on 90% Symbols List and the variable had the greatest influence on the mean cloze score. Since C had a negative coefficient, the number representing variable C needed to be decreased in order to increase the mean cloze

score. To decrease this figure, an attempt was made to include as many familiar mathematics terms and symbols as possible.

Variable D, number of question marks, was also negatively correlated with the mean cloze score. Therefore, the number of question marks was decreased in order to decrease the number representing variable D.

The instructional material used in Level A lessons was developed by manipulating Level B lessons with the aid of Kane's Formula II. This approach allowed the Level A lessons to be written at a lower or easier level of difficulty of reading. This same kind of technique was used to increase the reading difficulty for Level C lessons. These lessons appear in Appendix K. An attempt was made to increase the amount representing the variables with negative coefficients and to decrease the amount representing the variables with positive coefficients. The three mean cloze scores that were derived from this manipulation of variables are 31.5, 24.5, and 18.5. These data appear in Appendix L.

This manipulation procedure facilitated the writing of three units of three lessons each which could be referred to as having easy, medium, and difficult readability levels. The terms "easy," "medium," and "difficult" are, of course, relative terms, but Kane provides detailed instructions on ranking mean cloze scores and determining whether they are

indeed significantly different. This method is explained in Appendix A. Since the rating of the lessons between two different scorers could vary somewhat without causing two mean cloze scores to be significantly different, it did not seem necessary to have another person score the lessons.

A test covering the material in the three lessons was developed with the content taken directly from printed lessons. Again, the questions on the test were in line with the questions normally asked of this group of students in middle school. The test consisted of 26 items and was used both as the pretest and the posttest. This test appears in Appendix E.

Procedures

Figure 1 presents a flowchart of procedural activities.

1. In this study it was necessary to select the classes for collection of data before the development of the materials since the materials were built around the instructional material in the math class.

First, it was necessary that the school offer a separate program for regular mathematics. The school also had to provide an adequate population for the study.

A middle school in the Dallas/Fort Worth Metroplex met the criterion and the administration was willing to permit the investigator to participate in the study. After

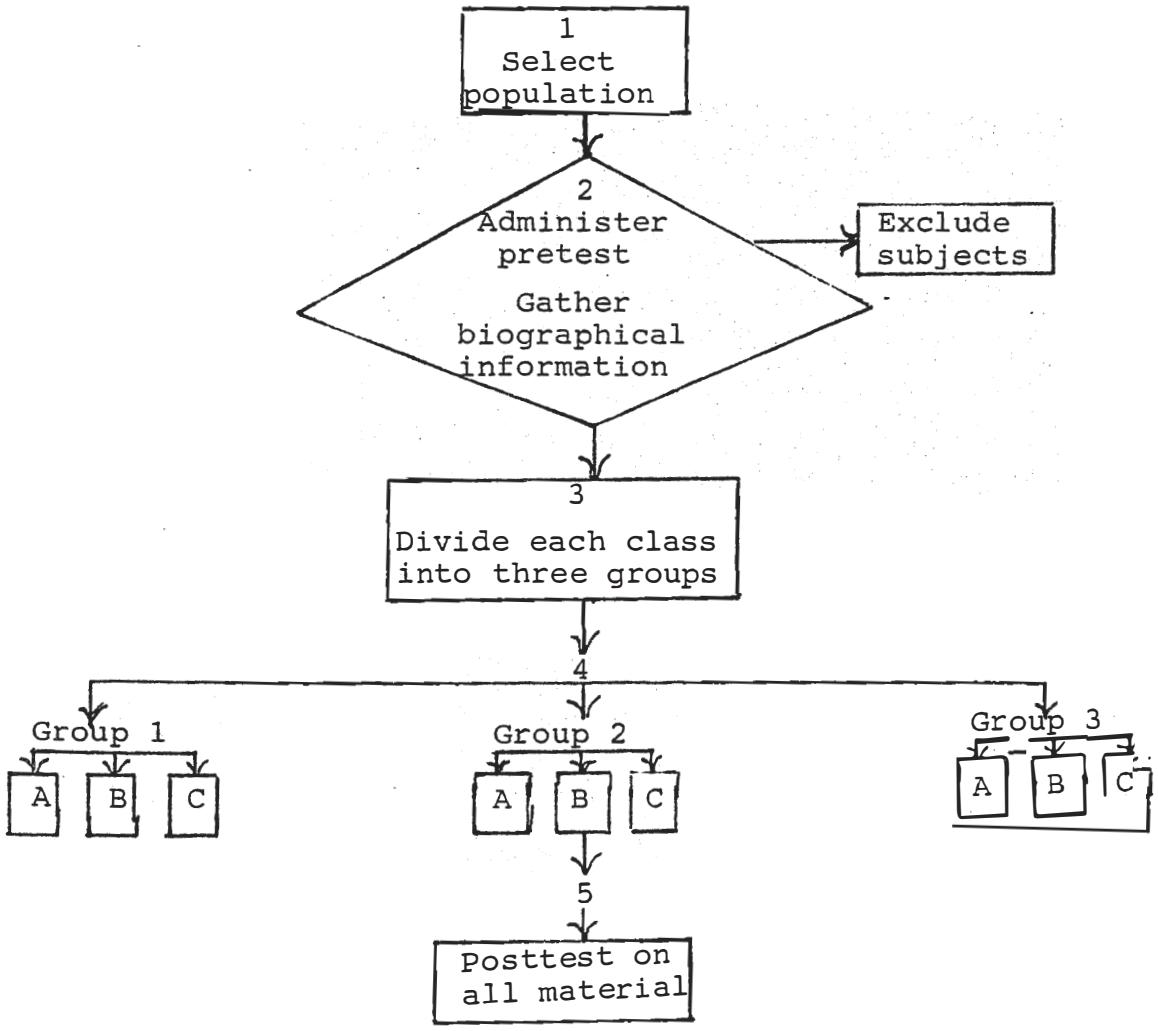


Figure 1. Procedures Flowchart

permission was secured to conduct the investigation, initial contacts revealed some of the limitations that were to be placed on the content of the lessons involved. The content of the material was to be taken from the curriculum. It was necessary to work closely with the faculty members concerned. The lessons were written with strict adherence to the request that the content be the same as was normally taught during this time of the semester.

2. The pretest/posttest (Appendix E) was administered to the students immediately preceding the instruction to determine the extent of his knowledge of the concept of sets.

The data were collected during the third week of the third six weeks spring semester. A full week of classes was allowed to complete the study. An hour was allotted for each lesson.

The class meeting was during the third and fourth week of the spring semester; therefore, the instructor took only a few minutes to check the roll and to make introductions. Since this was the first time most of the students had met the investigator, it was assumed that the group was not influenced by the attitudes of the investigator or the instructional techniques that were used. The instructor was not present during any of the remainder of the activities. The investigator administered the tests and distributed the lessons.

3. Since the students were not randomly assigned to the three sections, each section was divided into three random groups (group -A, group -B (control group), group -C to receive the three different levels of lessons. Three groups of random numbers were pre-arranged. The students were asked to number-off and were assigned the easy, medium, or difficult lesson according to which group their number was randomly placed. Each student was asked to write his name and number on the pretest, each lesson, and the posttest to provide a check to be certain that he had received the appropriate materials consistently.

4. Materials representing the three different levels of readability were given to the groups (Appendix F). Group A received the materials with the least difficult reading level, Group B (control group) received the materials with the medium difficult reading level, and Group C received the materials with the most difficult reading level. After a student had completed one lesson, he would turn it in and work the next lesson. Each student was allowed to proceed at his own pace. An hour, which should be ample time, was allowed for each lesson.

5. After a student had completed all three lessons, he was immediately given the pretest/posttest covering all of the material presented in the lessons. He was not required to wait for the other students. The posttest was the same

as the pretest. These results appear in Appendix G.

Subjects were given a brief explanation of the study; however, it was not emphasized that the material covered was part of their regular classwork. Also, it was explained that even though the results of their work during the five days would not directly affect their grade; that is, even though their teacher would not see the results of the post-test scores, the students would be tested over the material during their next regular exam with their instructor. Consequently, the question of motivation to the validity of the study may appear to be attenuated. At the end of the last session, the students who drew the medium or difficult lessons were given copies of the easy lessons for study at a later time, since they were to be tested on this material.

After the appropriate materials were passed out each day during the instructional days these were the instructions which were read to each group: (1) read carefully, (2) work all problems, (3) take notes on the material, (4) work independently. These were the instructions given for each group and lesson.

Analysis

Each section was divided into three random groups to receive the three different levels of lessons. Groups A, B, and C received the easy, medium, and difficult lessons

respectively. Easy, medium, and difficult lessons were determined by Kane's Formula II for predicting the mean cloze of a mathematical passage which was interpreted as readability level.

The treatment was assigned to groups randomly which made the groups about equal in size. A pretest and a post-test were administered over the content of the mathematics materials presented in the lessons (Appendix F).

The pretest/posttest consisted of 26 questions. Each correct response was given one point, yielding a maximum score of twenty-six.

The first analysis performed was a comparison of the group's low and high scores on the pretest and the posttest and their range. The second analysis was the variance of the difference on the pretest scores, the posttest scores, and the difference between the pretest and the posttest scores. In this study the .05 level of significance was used.

The computer center of the area metroplex school district was used in processing the data. ANOVA was used in the analysis.

Limitations

Some of the areas that were a limitation to this study were:

1. The population was small and it included only eighth grade students;

2. The time span of the study was five days;

3. The amount of variety of instruction;

4. The content of the lesson utilized in this study was chosen because of constraints imposed by the curriculum of the school participating in the study.

Definition of Terms

Word token denotes any graphic representation having a phoneme-grapheme relationship.

Math tokens are signs as +, -, and % which do not necessarily have a one-to-one phoneme-grapheme relation with spoken words.

Readability formula is a linear equation which predicts reading difficulty.

Variable is a symbol that holds a place for a numeral in a number sentence. (mathematical definition)

Predictor variable for a readability formula has two characteristics: it is highly related to the criterion measure of readability and it is relatively easy to quantify for a typical sample passage.

Regular eighth grade mathematics class is any student who scored between 6.4 and 9.5 grade placement on the Iowa Test of Basic Skills (ITBS).

Amount of learning is the increased knowledge gained between the time that a student takes the pretest and the posttest.

Readability is a measurable method of determining the difficulty of printed material.

Difficulty is something not easily done, accomplished, comprehended, or solved.

Cloze procedure is used as a measure of readability, of reading comprehension, and to assess reading difficulty.

Mean cloze is an average of cloze scores using a range of high, average, and low values for the predictor variables to determine precision for a wide range of possibilities.

CHAPTER IV

ANALYSIS

The purpose of this study was to determine the relationship between the readability level of mathematics materials on achievement for regular middle school students enrolled in mathematics. Regular mathematics students from a large inner-city school district comprised the population. Three classes of students enrolled in regular mathematics were included in the study. Eighteen of the students were excluded from the analysis because their ITBS scores did not meet the requirement for a regular eighth grade mathematics class. Thus, fifty-five subjects comprised the sample.

Each section was divided into three random groups to receive the three different levels of lessons. Groups A, B, and C received the easy, medium, and difficult lessons, respectively. Easy, medium, and difficult were determined by Kane's Formula II for predicting the mean cloze score of a mathematical passage which is interpreted as readability level.

The treatments were assigned to groups randomly which resulted in the following group-sizes: Group A (easy), 16; Group B (medium), 22; and Group C (difficult), 17. A pretest

and a posttest were administered over the content of the mathematics materials presented in the lessons (see Appendix F). The pretest and the posttest were the same and consisted of twenty-six questions. Each correct response was given one point, yielding a maximum score of twenty-six.

Data analysis in this study is presented graphically and in essay form. The procedural presentation of data is sequential from pretest to posttest. Two basic analyses were made of the data. First, a comparison of the range of pretest scores and posttest scores; second, analysis of variance of pretest scores, posttest scores, and the difference between the pretest score and the posttest score.

Arrangements were made with the school district's Control Data System, Serial: 209, Dist/B6700 MCP: System/MCP/FMlyinx002.32.360.1876, to process the data. The VRE library program was used. ANOVA was used to compute the analyses of variance.

The null hypothesis was, "There will be no significant difference in the amount of learning evidenced between groups with a given pretest score who received instruction from mathematics materials on varying levels of readability with the amount of learning varying inversely with the level of difficulty of reading." An analysis of variance was considered to be the most appropriate design for this study.

In Table 1 a comparison was made of the range of pretest scores and posttest scores between the three groups.

Table 1

Comparison of the Range of Pretest Scores and Posttest Scores

Groups	Pretest Scores Low	High	Range	Posttest Scores Low	High	Range
A (Easy)	0	21	20	8	24	15
B (Medium)	2	20	17	5	25	19
C (Difficult)	4	22	17	7	23	15

The range of scores was similar in each group on both the pretest scores and the posttest scores. Group A had the greatest range of pretest scores which was 20 while groups B and C had a pretest score range of 17. Group B had the greatest posttest score range of 19 while groups A and C had a range of 15.

In Table 2 the analysis involved the computation of the means. The means for groups A, B, and C were 6.9, 7.5, and 11.6, respectively, from a maximum score of 26. Since the pretest mean scores were so different, even though the groups were randomly grouped, a homogeneity of regression was run on the covariates to check for interaction on the posttest. There was no interaction between the covariates

Table 2

Analysis of Pretest Scores

Groups	Sum of Squares	Mean	F scores	df
A (Easy)	111.00	6.9	4.54	(1,15)
B (Medium)	164.00	7.5	4.32	(1,21)
C (Difficult)	197.00	11.6	4.49	(1,16)
Total Population	472.00	8.6	4.18	(1,54)

and the posttest. The results of homogeneity regression were .05. The probability of obtaining a difference this size is .78. Since this probability is greater than .05, the pretest scores were not significantly different. The results of the analysis of variance for the pretest scores are given in Table 2.

The results of the comparison of means and standard deviations of pretest scores and posttest scores are given in Table 3.

There was a statistically significant difference between the total population pretest mean scores and the total population posttest mean score which was 5.9 difference.

An analysis of variance was conducted on pretest score difference between the three groups. The results are presented in Table 4.

Table 3

Comparison of Means and Standard Deviations

of Pretest Scores and Posttest Scores

Groups	Pretest Mean	Pretest SD	Posttest Mean	Posttest SD	
A (Easy)	6.9	5.0	15.5	3.9	
B (Medium)	7.5	4.1	14.2	5.0	
C (Difficult)	11.6	5.7	13.8	5.1	
Total Population	8.6	5.2	14.5	4.7	5.9*

*p < .05

Table 4

Analysis of Variance on Pretest Scores

Source	Sum of Squares	Mean Squares	F Scores	df	p
Total Population	224.8721	112.4361	4.668	(2,54)	0.0137

Groups	Mean	Groups	Difference
A (Easy)	6.9	(A,B)	.6
B (Medium)	7.5	(A,C)	4.7
C (Difficult)	11.6	(B,C)	4.1

There was a statistically significant difference at the .05 level between the pretest scores for groups A and B and Groups A and C.

Table 5 presents the results of the analysis of variance on posttest score differences.

Table 5

Analysis of Variance on Posttest Scores

Source	Sum of Squares	Mean Squares	F Scores	df	p
Total Population	25.8930	12.9465	0.577	(2,54)	0.5654

Groups	Mean	Groups	Difference
A (Easy)	15.5	(A,B)	1.3
B (Medium)	14.2	(A,C)	1.7
C (Difficult)	13.8	(B,C)	.4

Since the p value was greater than .05, the difference between the groups on posttest scores was not significant.

The null hypothesis stated that there would be no difference in achievement because of readability levels. The results of the analysis of variance on pretest/posttest score differences which are in Table 6 reject the null hypothesis.

The information presented in Table 6 indicates a statistically significant difference at the .05 level between groups A and B and groups A and C on pretest/posttest scores. The null hypothesis was, therefore, rejected on the basis of these statistical findings.

Table 6

Analysis of Variance on Pretest/Posttest Difference

Source	Sum of Squares	Mean Squares	F Scores	df	p
Total Population	356.7491	178.3746	17.008	(2,54)	0.00000

Groups	Mean	Group	Difference
A (Easy)	8.6	(A,B)	1.9
B (Medium)	6.7	(A,C)	6.4
C (Difficult)	2.2	(B,C)	4.5

CHAPTER V

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

This study sought to determine the effect of the readability level of mathematics materials on achievement for urban middle school students enrolled in a regular mathematics class. The subjects were fifty-five students enrolled in three classes of regular mathematics at an urban middle school in the spring of 1981. Students were placed in these classes based upon their ITBS scores.

One mathematics lesson was written at three different levels of readability by manipulating the variables presented in Kane's Formula II (1974). Each class was randomly divided into three groups, each group being assigned a different level of readability. The students were administered a pretest and a posttest over the material presented in the lessons.

An analysis of variance was carried out using pretest scores, posttest scores, and the difference between pretest scores and posttest scores. These statistical analyses were reviewed in Chapter III.

The null hypothesis stated that there would not be a significant difference in the amount of learning evidenced between subjects who received instruction from mathematics

materials on varying levels of readability with the amount of learning varying inversely with the level of difficulty of reading. The group with the least difficult reading materials had a mean pretest score of 6.9 and a mean posttest score of 15.5. The maximum score was twenty-six. The group with the medium level of reading difficulty had a mean pretest score of 7.5 and a mean posttest score of 14.2. The group with the most difficult reading materials had a mean pretest score of 11.6 and a mean posttest score of 13.8 (see Table 3). The probability of the obtained F in posttest scores for the three groups reached .0000. Based on these results, the null hypothesis was rejected.

Conclusions

The results of the analyses of this study suggest that readability of mathematics materials is a significant variable in student achievement in regular eighth grade mathematics. If additional studies are carried out which verify this relationship, the instructor of regular eighth grade mathematics may be directly affected. He may be forced to consider the readability level of the mathematics materials he selects for his students. Because individualized instruction is being used more frequently in math programs, the student would be expected to read more of the material on his own in order to progress at his own pace. Materials

would need to be written at the appropriate level of readability and using appropriate concepts in order for the student to profit from this method of instruction.

Textbook selection committees are another group who could effectively use a readability formula for mathematics materials. By applying a formula, grossly inadequate materials could be eliminated quickly from further consideration.

Textbook publishing companies would be affected ultimately by the focus on readability of mathematics materials. If teachers demand materials written on appropriate levels of readability, correct level of concepts and textbook committees reject a company's book because of these, the publisher surely would respond.

Because the results of this study have significant implications for the future of mathematics programs, further research definitely needs to be conducted.

Recommendations

In future research, consideration should be given to the following:

1. The content of the lessons used in this study was selected because of constraints imposed by the curriculum of the school participating in this study. Because the study involved only one mathematical concept, the results could not be generalizable to other areas of content without further

investigation. The effects of readability and difficulty of concepts of materials may either increase or decrease student achievement, therefore, readability and concept should be controlled.

2. The readability formula used in this study was validated for use with mathematics written for grades six through ten. The content of materials used in a regular eighth grade mathematics class probably would not vary from that presented in grades six through ten, especially because it is a controlled placement. A study intended to validate the formula with regular eighth grade students at the middle school level would strengthen the application of the formula to materials used with this group of students.

3. The school selected for this particular study did not use individualized instruction in its regular mathematics program; however, because this form of instruction is the trend, a study should be designed around lessons which are in the form of self-instructional units.

4. Although students were randomly assigned to their groups, there was a variation in pretest scores. Because there was a difference in the pretest scores, a study concentrating on a relatively unfamiliar topic with more consistent pretest scores might prove revealing. The posttest scores perhaps would more accurately reflect what the students had learned rather than the amount of review that they

needed.

5. This study was conducted during the spring semester. The study needs to be replicated during other times of the year.

6. The results of this study may not be generalizable to other populations because:

(a) The size of the population was small and it included only eighth grade students. This study needs to be replicated on a larger sample and with a different population in order to produce more conclusive results.

(b) The amount of instruction was small. It covered only five days. The effect of the readability level of lessons given over a longer period of time needs to be considered.

(c) Lessons should be written so that the only variables are those included in Kane's Formula II. Kane originally defined and studied 110 potential variables to be used in his readability formula. He eliminated the variables that did not have sufficient power to predict the mean cloze score. It seems to follow that any variations other than in the variables accounted for in the formula would not significantly alter the posttest scores. However, a tightly controlled study altering only the variables in the formula would verify this point.

(d) The content of the lessons used in this study was selected because of constraints imposed by the curriculum of the school participating in the study. Other topics which include unfamiliar content should be researched to test the readability treatment rather than a review of content.

APPENDICES

APPENDIX A

PROCEDURES FOR USING KANE'S FORMULA II

The following instructions were abstracted from Kane, R. B., Byrne, M. A., and Hater, M. A., Helping Children Read Mathematics, New York: American Book Company, 1974. Kane presents two readability formulas in his book Formula II is the more sophisticated and was used in this study. Since the readability of a book was not being measured, the procedures had to be altered slightly. Rather than taking a sample of 400 tokens, all of the 400 token sets in the lessons were included in the count. The mean count of each variable was used in the formula.

Procedures for Using Formula II for the Language of Mathematics

A. How many samples to select from a book.

It is important that enough samples be selected to represent the material being evaluated. To insure sufficient sampling follow this procedure.

A 400-token sample should be selected in every 30 pages from a mathematics book except when the number of pages in the book is less than 300.

Exclude the table of contents, glossary, and other similar front and back material from the total page count. If the number of pages in a book is less than 300, divide the number by 10. The quotient tells the interval from which a selection be made.

Example: There are 220 pages in a book. Since 220/10 = 22, select a 400-token sample in every 22 pages. This method will assure a minimum of 10 samples from a book and at least one sample from every 30 pages.

B. How to select a 400-token sample.

Sample pages should not be handpicked for in so doing you may select passages which are easy, or hard, or all on one topic, etc. Instead, the location of each sample should be randomly selected. For brief articles of a few pages, the formula could be used on the first 400 tokens.

C. What tokens to count.

Begin the sample with a title. Count the title and all words and symbols on the page. Include: (a) numerals and letters for ordering, (b) words or symbols on graphs, (c) symbols for figures, (d) placeholders, and (e) any punctuation symbol when used with a special meaning in mathematics. Do not count pictures or arrows, geometric figures, and punctuation which does not have special mathematical meaning.

D. What are tokens?

Word tokens: Most word tokens are simply written words. Example: "number", "the", "follow", and "triangle". In general a word token is separated from surrounding material by spaces. The number of word tokens in an abbreviation is determined by the number of different word tokens which the

abbreviation replaces which have some representation in the abbreviated form. "U.S.A." has three word tokens; "cm." has one word token. Hyphenated word tokens are counted as one or more word tokens according to whether the parts can be used alone with meaning. "Non-linear" is one word token; "one-to-one" is three word tokens.

Math tokens: These are signs which appear in the language of mathematics which are not word tokens, punctuation, or drawings such as 2,+,=, and %. They are the smallest units which can be used independently to convey the intended meaning of that part of the written material. These rules may be helpful: (1) A graphic sign in which all parts are connected is at most one math token. For example, "x" is one math token. (2) A graphic sign in which all parts are not connected is more than one math token if two or more parts independently convey the intended meaning of part of the material or if the parts are separated by other tokens. For example: = and $\neq$ are each one math token; $\geq$, x^2, 35 are each two math tokens; (a) is three math tokens.

E. Order in which tokens are counted.

Tokens when written on a math page are not always ordered from left to right as in ordinary English. The order imposed by vocalization should be followed. A vocalization is not necessarily unique but it should be consistent across uses of the formula on passages that will be compared.

The expression $X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ has tokens ordered as follows: x,=,-,b,+,-,$\sqrt{\ }$,b,2,-,4,a,c,-,2,a.

Directions for Measuring Variables on Each 400-Token Sample

Counting variable A: The number of words not on the Dale List of 3000 Familiar Words that are not on the List of Mathematics Words Familiar to 7th-8th grade students.

1. Circle all word tokens in the sample that are not on the Dale List of 3000 Familiar Words. Circle all words even if they occur more than once. Do not consider numerals or symbols in this count. The Dale List and directions for suing it are found in Appendix F.

2. Consider each of the circled words. If a word is not on the List of Familiar Mathematics Words, put a line through the word. The List of Familiar Mathematics Words is given in Appendix G.

3. Count the words that have both a circle and a line drawn through them. This count is variable A.

Counting variable B: The number of changes from a word token to a math token and vice-versa. The count of variable B in the example below would be 7.

Example: In $\downarrow$ ABC, $\downarrow$ if $\downarrow$ B = 45° $\downarrow$ and $\downarrow \angle C = 90°$, how many degrees are then in $\downarrow \angle A$?

Note: The arrows shown in this example indicate changes from word to math token or vice-versa, and are not part of the original passage.

A figure should be considered in itself. That is, only count changes that occur in the figure itself. There is no count at the beginning or ending of a figure.

Counting variable C: The number of different mathematics words on the List of Mathematics Words Not Familiar to 7th-8th grade students plus the number of different mathematics symbols not on the List of Mathematics Symbols Known by 7th-8th grade students.

Appendix H contains the List of Unfamiliar Mathematics Words. Appendix I contains the List of Familiar Mathematics Symbols.

For each sample:

1. Count the number of word tokens in the sample that are on the List of Unfamiliar Mathematics Words found in Appendix H. Directions for making the count are in Appendix H.

Count the number of these word tokens that are different. That is, if "commutative" were used four times in the sample it would only count as one unfamiliar word in this count. However, if different word forms are used they are counted as different word tokens in this count. This is count C_1.

Count the number of math tokens in the sample that are not on the List of Familiar Mathematics Symbols found in Appendix I.

Count the number of these mathematics tokens that are different. A symbol is considered familiar only if it is in a context similar to that given in Appendix I. This is count C_2. The sum of variables C_1 and C_2 is equal to C.

Counting variable D: The number of question marks. Merely count the number of "?" in the sample and use in the formula.

APPENDIX B

APPLICATION OF DALE-CHALL READABILITY FORMULA

The following tables give the results of the application of the Dale-Chall readability formula to the three levels of lessons. There are three-one hundred word samples taken from each level. Each level of lesson A, B, and C was divided into three lessons and a one hundred word sample was taken from each lesson.

Level A Lesson

(Easy)

		Predictor Variable	
		X	Y
Set of	1	12	12.5
100	2	7	12.5
Sample	3	12	14.29
Total		31	39.29
Mean		10.33	13.10

R = reading-grade score of a pupil who could answer correctly 50% of the questions on a reading comprehension test covering the passage

$R = .1579X + .0496Y + 3.6365$

$R = .1579(10.33) + .0496(13.10) + 3.6365$

$R = 1.63 + .65 + 3.6365$

$R = 5.92$

$R = 5.9$

Level B Lesson

(Medium)

		Predictor Variable	
		X	Y
Set of	1	28	20
100	2	10	16.6
Sample	3	21	14.29
	Total	59	50.89
	Mean	19.67	16.96

R = reading-grade score of a pupil who could answer correctly 50% of the questions on a reading comprehension test covering the passage

$R = .1579X + .0496Y + 3.6365$

$R = .1579(19.67) + .0496(16.96) + 3.6365$

$R = 3.11 + .84 + 3.6365$

$R = 7.59$

$R = 7.6$

Level C Lesson

(Difficult)

		Predictor Variable	
		X	Y
Set of	1	22	20
100	2	20	25
Samples	3	16	16.6
	Total	58	61.6
	Mean	19.3	20.5

R = reading-grade score of a pupil who could answer correctly 50% of the questions on a reading comprehension test covering the passage

$R = .1579X + .0496Y + 3.6365$

$R = .1579(19.3) + .0496(20.5) + 3.6365$

$R = 3.05 + 1.02 + 3.6365$

$R = 7.71$

$R = 7.7$

APPENDIX C

ESTIMATING THE PRECISION OF KANE'S FORMULA II*

Estimating the Precision of Kane's Formula II*

We need a way to determine whether predicted reading scores in two different books indicate different reading levels or are in fact due to errors in measurement. Error is inherent in all measurement; predicting reading difficulty is no exception. We examined several hundred combinations of high, average, and low values for the predictor variables to determine precision for a wide range of possibilities. As a result of these findings, selections were made which would allow readers to compare different cloze scores with some precision while keeping the amount of work involved in calculation to a minimum. An illustrative example of how to determine whether two predicted cloze scores are far enough apart to represent a true difference is included. The smallest error occurs when each predictor variable is replaced by the mean tally for that variable over the 70 cases used to generate Formula II. As deviations from these means occur the error increases.

Table 1

Means and Standard Deviations

for the Predictor Variables

	A	B	C	D
Mean (M)	27.8	50.7	11.5	4.8
Standard Deviation (SD)	14.2	23.2	4.8	6.5
$M \pm 0.5$ SD	34.9 to 20.7	62.3 to 39.1	13.9 to 9.1	8.1 to 1.5
Between 0.5 SD and 1.5 SD	49.1 to 34.9 20.7 to 6.5	85.5 to 62.3 39.1 to 15.9	18.7 to 13.9 9.1 to 4.3	14.6 to 8.1 1.5 to 0.0
Between 1.5 SD and 3.0 SD	70.4 to 49.1 6.5 to 0.0	120.3 to 85.5 15.9 to 0.0	25.9 to 18.7 4.3 to 0.0	24.3 to 14.6

The table below lists the amounts to add to and subtract from a predicted cloze score to mark the range of true cloze scores. Select the combination of predictor variables scores which most closely fits the scores you tallied for the passage being rated. Next select the confidence level, .90 or .75, you wish to use. A level of .90 means that 90% of all possible true scores lie within the given range.

Table 2

Confidence Limits for Predicted Cloze Scores

Description of Predictor Variables	Confidence Level .90	.75
1. All within 0.5 SD of mean	1.5	1.1
2. Three within 0.5 SD of mean, one between 0.5 and 1.5 SD of the mean	2.0	1.6
3. Three within 0.5 SD of mean, one between 1.5 and 3.0 SD of the mean	3.9	2.6
4. Two within 0.5 SD of mean, two between 0.5 and 1.5 SD of the mean	2.3	1.8
5. Two within 0.5 SD of mean, two between 1.5 and 3.0 SD of the mean	4.7	2.9
6. One within 0.5 SD of mean, three between 0.5 and 1.5 SD of the mean	2.7	2.0
7. One within 0.5 of mean, three between 1.5 and 3.0 SD of the mean	5.4	3.5
8. All between 0.5 and 1.5 SD of the mean, and all on same side of mean	2.6	1.9
9. All between 0.5 and 1.5 SD of the mean; three on one side, one on opposite side of the mean	2.8	2.3
10. All between 0.5 and 1.5 SD of the mean; two on each side of the mean	3.1	2.6
11. Three between 0.5 and 1.5 SD of the mean, one between 1.5 and 3.0 SD of the mean	4.2	3.3
12. Two between 0.5 and 1.5 SD of the mean, two between 1.5 and 3.0 SD of the mean	5.3	3.5
13. One between 0.5 and 1.5 SD of the mean, three between 1.5 and 3.0 SD of the mean	5.7	3.6
14. All between 1.5 and 3.0 SD above the mean	5.7	3.6
15. All between 1.5 and 3.0 SD of the mean; three above and one below the mean	6.0	3.9
16. All between 1.5 and 3.0 SD of the mean; two above and two below the mean	6.0	4.0

Example: The variable tallies and predicted cloze scores for two passages are given below.

	A	B	C	D	Predicted Cloze Score
Passage 1	22	60	10	2	33.7
Passage 2	42	74	16	1	29.7

For Passage 1, the predictor variables are all within 0.5 standard deviations of their respective means (See Table 1). Thus we refer to line 1 of the table of confidence limits and combine the scores of 1.5 and 1.1 with the mean of 33.7. Note that we may be 90% confident that the true cloze score for Passage 1 is 33.7 ± 1.5 and 75% confident that it is 33.7 ± 1.1. For Passage 2 all the predictor variables are between 0.5 and 1.5 standard deviations of their respective means. Moreover, three of them are above their respective means while one, variable D, is below its mean. Therefore we refer to line 9 of the table. We may be 90% confident that the true cloze score for passage 2 is 29.7 ± 2.8 and 75% confident that it is 29.7 ± 2.3. These numbers are calculated for you and appear in the following table.

Ranges of True Cloze Scores

Level	Passage 1	Passage 2
90%	32.2 - 35.2	26.9 - 32.5
75%	32.6 - 34.8	27.4 - 32.0

From the table above, we observe that at the 90% level the true cloze scores do not need to be different since the ranges overlap. As an example, both passages could have a score of 32.4. Using the 75% confidence limits we note that the intervals do not overlap since 32.6 is greater than scores for these passages are different.

*Quoted direct from Helping Children Read Mathematics.

APPENDIX D

INFORMATION SHEET

INFORMATION SHEET

Name_ _____ Age_____

Sex _____ Ethnic origin_____

Score on ITBS-Math_____(grade level)

Score on ITBS-Reading_____(grade level)

APPENDIX E

DATA EMPLOYED IN THE STUDY

Data Employed in the Study

Subjects		Level	Pretest	Posttest
1		C	19	19
2		B	08	14
3		A	05	15
4		C	06	12
5		C	06	07
6	omitted	A	07	
7		C	11	13
8		B	09	14
9		A	00	17
10		C	13	15
11		C	04	08
12	omitted	A		
13		B	10	17
14		C	14	14
15	omitted	A		
16		A	04	16
17		B	04	08
18		A	06	16
19		A	05	15
20		B	09	16
21		B	05	05
22		A	05	13
23		A	04	18
24		C	11	12
25		B	08	12
26		B	09	09
27		B	04	10
28		B	08	10
29	omitted	C		
30		B	07	16
31		C	06	07
32		B	07	15
33		B	02	12
34	omitted	C	06	08
35		A	06	08
36		A	07	16
37	omitted	A		
38	omitted	A	06	13
39		C	10	13
40		A	09	14

Subjects		Level	Pretest	Posttest
41		B	04	10
42		B	12	23
43	omitted	B	07	10
44		B	04	14
45	omitted	C	05	05
46	omitted	A	03	
47	omitted	A	02	18
48		A	07	11
49	omitted	C	04	05
50	omitted	B	05	
51	omitted	C		
52	omitted	C	01	07
53		B	05	16
54		C	09	09
55		C	11	15
56		B	02	09
57		A	04	12
58		A	16	22
59		B	13	21
60		B	09	19
61		A	04	14
62		C	10	12
63		C	22	22
64		B	05	17
65		A	21	24
66		B	20	25
67	omitted	A		
68	omitted	A		
69	omitted	A	10	10
70	omitted	C		
71		A	08	17
72		C	05	12
73		C	20	23
74		C	20	22

Demographic Variables and ITBS Math and Reading Scores

Subjects		Age	Sex	Ethnic*	Math ITBS	Reading ITBS
01		14	F	A	8.7	8.5
02		14	F	A	9.4	9.0
03		14	F	B	7.3	6.4
04		14	F	B	7.7	7.4
05		14	F	B	9.3	7.3
06	omitted	14	F	B		
07		14	F	B	8.6	8.5
08		14	F	B	7.1	8.1
09		14	M	M	8.6	8.6
10		14	F	B	9.2	8.1
11		14	F	B	8.2	8.7
12	omitted	14	F	A		
13		14	F	A	7.4	6.2
14		14	F	A	8.1	6.5
15	omitted	14	M	B		4.5
16		13	F	A	8.7	11.2
17		13	F	B	6.6	3.3
18		13	F	B	8.7	9.6
19		14	F	A	8.4	9.4
20		13	F	A	8.3	8.7
21		13	F	B	6.5	6.2
22		14	F	A	8.1	8.0
23		14	F	A	8.5	8.6
24		15	M	M	7.0	7.3
25		14	F	B	6.8	4.0
26		13	F	A	8.3	6.3
27		14	F	B	7.3	7.2
28		14	M	B	6.6	4.4
29	omitted	14	M	A		
30		15	M	A	6.8	4.5
31		14	M	B	7.9	7.3
32		14	F	B	6.9	8.4
33		13	F	B	6.7	7.3
34	omitted	14	M	A	6.3	4.0
35		15	F	A	8.4	8.8

Subjects		Age	Sex	Ethnic*	Math ITBS	Reading ITBS
36		14	F	A	7.3	7.8
37	omitted	15	M	A		
38	omitted	13	F	B		
39		14	M	B	8.8	6.2
40		14	F	B	9.2	8.3
41		14	F	B	8.0	9.2
42		13	F	A	9.3	10.0
43	omitted	15	M	B	5.3	5.5
44		13	M	A	6.6	6.8
45	omitted	15	M	B	4.5	3.5
46	omitted	13	M	A		
47	omitted	13	M	M	5.1	3.7
48		13	F	A	6.6	6.4
49	omitted	13	F	A	5.1	3.2
50	omitted	14	M	B	5.8	4.6
51	omitted	14	M	A		
52	omitted	13	M	A	6.3	
53		14	M	A	9.0	6.9
54		13	M	B	8.5	6.4
55		13	F	A	9.4	7.8
56		13	M	A	9.3	9.8
57		14	M	A	7.4	8.5
58		14	M	A	9.5	10.5
59		14	F	B	8.8	11.2
60		13	F	A	9.3	9.9
61		14	M	A	8.1	8.0
62		14	F	B	8.6	9.5
63		14	F	A	9.3	13.0
64		14	M	A	8.7	8.6
65		14	F	A	9.5	13.2
66		14	F	B	9.1	12.9
67	omitted	14	M	B		
68	omitted	13	F	B		
69	omitted	15	M	A	5.4	5.2
70	omitted	13	M	A		
71		14	F	A	8.6	9.7
72		14	F	A	9.2	10.8
73		14	F	A	9.5	12.3
74		14	F	A	8.8	12.2

APPENDIX F

PRETEST-POSTTEST

TEST

1. Given X = {1,2,3,4} Y = {2,4,6} Z {5,6,7,8}

(a) X U Z (a) _____

(b) X ∩ ø (b) _____

(c) X ∩ Z (c) _____

(d) Y U ø (d) _____

(e) Y ∩ Z (e) _____

(f) X ∩ Y (f) _____

(g) X U Y (g) _____

2. Given in set builder notation describe the following sets by the enumeration method.

(a) A = {x | x principals of our school} (a) _____

(b) B = {x | x odd whole numbers} (b) _____

(c) C = {x | x < 5 counting numbers} (c) _____

3. Which of the following are finite and which are infinite?

(a) {2,4,6,8,...} (a) _____

(b) {5,10,15,20,.....,95} (b) _____

(c) {All of the fish in the sea} (c) _____

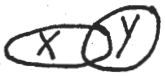
(d) {The set of even numbers} (d) _____

4. Tell which of the following are disjoint sets: (yes or no)

(a) $X = \{6,7,8,9\}$ $Y = \{1,2,3,4\}$ (a) _____

(b) $W = \{2,3,6,...\}$ $Z = \{3,6,9,...\}$ (b) _____

5. Shade in the indicated area:

$X \cap Y$ (b) $X \cup Y$ 

(c) $X \cup Y$ (d) $X \cap Y$

6. Give an appropriate universal set for the following set of numbers:

$N = \{3,4,5\}$ _____

7. Answer the following true or false

(a) $\{2,4,6\} = \{3-1,\ 4+2,\ 2+2\}$ (a) _____

(b) $\{2,3,6\} \subset \{1,2,3,4,5,6\}$ (b) _____

(c) $\{3,4,5\} \subset \{3,4,5\}$ (c) _____

(d) $\{1,2,3\} \subset \{1,2,3,4\}$ (d) _____

(e) $2 \in \{2,4\}$ (e) _____

APPENDIX G

PROCEDURE FOR USING THE DALE LIST OF 3000 EASY WORDS*

Procedure for Using the Dale List of 3000 Easy Words*

Directions for counting the number of word tokens not on the Dale List of 3000 Familiar Words.

In making this count, special rules are necessary for common and proper nouns, verbs and other parts of speech. these are given in the section which follows.

A. Common nouns:

1. Consider familiar all regular plurals and possessives of words on the list; "boys" is familiar because "boy" is on the list (plural by adding "s"); "girl's" is familiar because "girl" is on the list (possessive); "churches" is familiar because "church" is on the list (plural by adding "es"); "armies" is familiar because "army" is on the list (plural by changing "y" to "ies").

2. Count irregular plurals as unfamiliar, even if the singular form appears on the list; "oxen" is unfamiliar, although "ox" is on the list. Several irregular plurals, however, are listed on the word list.

3. Count as unfamiliar a noun that is formed by adding "er" or "r" to a noun or verb appearing on the word list unless this form is indicated on the list.

B. Proper nouns:

1. Names of persons and places are considered familiar. "Japan" is familiar even though it does not

appear on the word list.

2. Abbreviations

a. In counting the words in a sample, an abbreviation is counted as one word. "Y.M.C.A." is counted as one word. "Nov." is counted as one word.

b. In making the unfamiliar word count, an abbreviation is counted as one unfamiliar word only. "Y.M.C.A." is considered one unfamiliar word. "Nov." is considered familiar.

C. Verbs:

Consider familiar the third-person, singular forms ("s" or "ies" from "y"), present-participle forms ("ing"), past-participle forms ("n"), and past-tense forms ("ed" or "ied" from "y"), when these are added to verbs appearing on the list. The same rule applies when a consonant is doubled before adding "ing" or "ed".

D. Adjectives:

1. Comparatives and superlatives of adjectives appearing on the list are considered familiar. The same rule applies if the consonant is doubled before adding "er" or "est".

2. Adjectives formed by adding "n" to a proper noun are familiar. For example, "American", "Austrian".

3. Count as unfamiliar an adjective that is formed by adding "y" to a word that appears on the list.

But consider the word familiar if "y" appears in parentheses following the word.

E. Adverbs:

1. Consider adverbs familiar which are formed by adding "ly" to a word on the list. In most cases "ly" will be indicated following the word.

2. Count as unfamiliar words which add more than "ly", like "easily".

F. Miscellaneous special cases:

1. Words formed by adding "en" to a word on the list (unless otherwise indicated on the list) are considered unfamiliar.

2. Count a word unfamiliar if two or more endings are added to a word on the list; "clippings" is considered unfamiliar, although "clip" is on the list.

3. Words on the list to which "-tion", "-ation", and other suffixes not previously mentioned are unfamiliar.

Dale List of 3000 Familiar Words

a
able
aboard
about
absent
accept
accident
account
ache (ing)
acorn
acre
across
act (s)
add
address
admire
adventure
afar
afraid
after
afternoon
afterward (s)
again
against
age
aged
ago
agree
ah
ahead
aid
aim
air
airfield
airplane
airport
airship
airy
alarm
alike
alive
all
alley
alligator
allow
almost
alone
along
aloud
already
also
always
am
America
American
among
amount
an
and
angel
anger
angry
animal
another
answer
ant
any
anybody
anyhow
anyone
anything
anyway
anywhere
apart
apartment
ape
apiece
appear
apple
april
apron
are
aren't
arise
arithmetic
arm
armful
army
arose
around
arrange
arrive (d)
arrow
art
artist
as
ash (es)
aside
ask
asleep
at
ate
attack
attend
attention
August
aunt
author
auto
automobile
autumn
avenue
awake (n)
away
awfully
awhile
ax
baa
babe
baby (ies)
back
background
backward (s)

bacon
bad (ly)
badge
bag
bake (r)
bakery
baking
ball
balloon
banana
band
bandage
bang
banjo
bank (er)
bar
barber
bare (ly)
barefoot
bark
barn
barrel
base
baseball
basement
basket
bat
batch
bath
bathe
bathing
bathroom
bathtub
battle
battleship
bay
be (ing)
beach
bead
beam
bean
bear
beard
beast
beat (ing)
beautiful
beautify
beauty
became
because
become
becoming
bed
bedbug
bedroom
bedspread
bedtime
bee
beech
beef
beefsteak
beehive
been
beer
beet
before
beg
began
beggar
begged
begin
beginning
begun
behave
behind
believe
bell
belong
below
belt
bench
bend
beneath
bent
berry (ies)
beside (s)
best
bet
better
between
bib
bible
bicycle
bid
big (ger)
bill
billboard
bin
bind
bird
birth
birthday
biscuit
bit
bite
bitting
bitter
black
blackberry
blackbird
blackboard
blackness
blacksmith
blame
blank
blanket
blast
blaze
bleed
bless
blessing
blew
blind (s)
blindfold
block
blood
bloom
blossom
blot

blow
blue
blueberry
bluebird
bluejay
blush
board
boast
boat
bob
bobwhite
body (ies)
boil (er)
bold
bone
bonnet
boo
book
bookcase
bookkeeper
boom
boot
born
borrow
boss
both
bother
bottle
bottom
bought
bounce
bow
bowl
bow-wow
box (es)
boxcar
boxer
boy
boyhood
bracelet
brain
brake
bran
branch
brass
brave
bread
break
breakfast
breast
breath
breathe
breeze
brick
bride
bridge
bright
brightness
bring
broad
broadcast
broke (n)
brook
broom
brother
brought
brown
brush
bubble
bucket
buckle
bud
buffalo
bug
buggy
build
building
built
bulb
bull
bullet
bum
bumblebee
bump
bun
bunch
bundle
bunny
burn
burst
bury
bus
bush
bushel
business
busy
but
butcher
butt
butter
buttercup
butterfly
buttermilk
butterscotch
button
buttonhole
buy
buzz
by
bye
cab
cabbage
cabin
cabinet
cackle
cage
cake
calendar
calf
call (er) (ing)
came
camel
camp
campfire
can
canal
canary
candle
candlestick

candy
cane
cannon
cannot
canoe
can't
canyon
cap
cape
capital
captain
car
card
cardboard
care
careful
careless
carelessness
carload
carpenter
carpet
carriage
carrot
carry
cart
carve
case
cash
cashier
castle
cat
catbird
catch
catcher
caterpillar
catfish
catsup
cattle
caught
cause
cave
ceiling
cell
cellar
cent
center
cereal
certain (ly)
chain
chair
chalk
champion
chance
change
chap
charge
charm
chart
chase
chatter
cheap
cheat
check
checkers
cheek
cheer
cheese
cherry
chest
chew
chick
chicken
chief
child
childhood
children
chill (y)
chimney
chin
china
chip
chipmunk
chocolate
choice
choose
chop
chorus
chose (n)
christen
Christmas
church
churn
cigarette
circle
cirus
citizen
city
clang
clap
class
classmate
classroom
claw
clay
clean (er)
clear
clerk
clever
click
cliff
climb
clip
cloak
clock
close
closet
cloth
clothes
clothing
cloud (y)
clover
clown
club
cluck
clump
coach
coal
coast
coat

cob
cobbler
cocoa
coconut
cocoon
cod
codfish
coffee
coffeepot
coin
cold
collar
college
color (ed)
colt
column
comb
come
comfort
comic
coming
company
compare
conductor
cone
connect
coo
cook (ed)
cook (ing)
cooky (ie) (s)
cool (er)
coop
copper
copy
cord
cork
corn
corner
correct
cost
cot
cottage
cotton
couch
cough
could
couldn't
count
counter
countries
country
county
course
court
cousin
cover
cow
coward (ly)
cowboy
cozy
crab
crack
cracker
cradle
cramps
cranberry
crank (y)
crash
crawl
crazy
cream (y)
creek
creep
crept
cried
croak
crook (ed)
crop
cross (ing)
cross-eyed
crow
crowd (ed)
crown
cruel
crumb
crumble
crush
crust
cry (ies)
cub
cuff
cup
cupboard
cupful
cure
curl (y)
curtain
curve
cushion
custard
customer
cut
cute
cutting
dab
dad
daddy
daily
dairy
daisy
dare
damage
dame
damp
dance (r)
dancing
dandy
danger (ous)
dare
dark (ness)
darling
darn
dart
dash
date
daughter
dawn
day
daybreak

daytime
dead
deaf
deal
dear
death
December
decide
deck
deed
deep
deer
defeat
defend
defense
delight
den
dentist
depend
deposit
describe
desert
deserve
desire
desk
destroy
devil
dew
diamond
did
didn't
die (d) (s)
difference
different
dig
dim
dime
dine
ding-dong
dinner
dip
direct
direction
dirt (y)
discover
dish
dislike
dismiss
ditch
dive
diver
divide
do
dock
doctor
does
doesn't
dog
doll
dollar
dolly
done
donkey
don't
doorbell
doorknob
doorstep
dope
dot
double
dough
dove
down
downstairs
downtown
dozen
drag
drain
drank
draw (er)
draw (ing)
dream
dress
dresser
dressmaker
drew
dried
drift
drill
drink
drip
drive (n)
driver
drop
drove
drown
drowsy
drug
drum
drunk
dry
duck
due
dug
dull
dumb
during
dust (y)
duty
dwarf
dwell
dwelt
dying
each
eager
eagle
ear
early
earn
earth
east (ern)
easy
eat (en)
edge
egg
eh
eight
eighteen
eighth

eighty
either
elbow
elder
eldest
electric
electricity
elephant
eleven
elf
elm
else
elsewhere
empty
end (ing)
enemy
engine
engineer
English
enjoy
enough
enter
envelope
equal
erase (r)
errand
escape
eve
even
evening
ever
every
everybody
everyday
everyone
everything
everywhere
evil
exact
except
exchange
excited
exciting
excuse
exit
expect
explain
extra
eye
eyebrow
fable
face
facing
fact
factory
fail
faint
fair
fairy
faith
fake
fall
false
family
fan
fancy
far
faraway
fare
farmer
farm (ing)
far-off
farther
fashion
fast
fasten
fat
father
fault
favor
favorite
fear
feast
feather
February
fed
feed
feel
feet
fell
fellow
felt
fence
fever
few
fib
fiddle
field
fife
fifteen
fifth
fifty
fig
fight
figure
file
fill
film
finally
find
fine
finger
finish
fire
firearm
firecracker
fireplace
fireworks
firing
first
fish
fisherman
fist
fit (s)
five
fix
flag
flake
flame

flap
flash
flashlight
flat
flea
flesh
flew
flies
flight
flip
flip-flop
float
flock
flood
floor
flop
flour
flow
flower (y)
flutter
fly
foam
fog
foggy
fold
folks
follow (ing)
fond
food
fool
foolish
foot
football
footprint
for
forehead
forest
forget
forgive
forgot (ten)
fork
form
fort
forth
fortune
forty
forward
fought
found
fountain
four
fourteen
fourth
fox
frame
free
freedom
freeze
freight
French
fresh
fret
Friday
fried
friend (ly)
friendship
frighten
frog
from
front
frost
frown
froze
fruit
fry
fudge
fuel
full (y)
fun
funny
fur
furniture
further
fuzzy
gain
gallon
gallop
game
gang
garage
garbage
garden
gas
gasoline
gate
gather
gave
gay
gear
geese
general
gentle
gentleman
gentlemen
geography
get
getting
giant
gift
gingerbread
girl
give (n)
giving
glad (ly)
glance
glass (es)
gleam
glide
glory
glove
glow
glue
go (ing)
goes
goal
goat
gobble
god (G)
godmother
gold (en)

goldfish
golf
gone
good (s)
good-by (bye)
good-looking
goodness
goody
goose
gooseberry
got
govern
government
gown
grab
gracious
grade
grain
grand
grandchild
grandchildren
granddaughter
grandfather
grandma
grandmother
grandpa
grandson
grandstand
grape (s)
grapefruit
grass
grasshopper
grateful
grave
gravel
graveyard
gravy
gray
graze
grease
great
green
greet
grew
grind
groan
grocery
ground
group
grove
grow
guard
guess
guest
guide
gulf
gum
gun
gunpowder
guy
ha
habit
had
hadn't
hail
hair
haircut
hairpin
half
hall
halt
ham
hammer
hand
handful
handkerchief
handle
handwriting
hang
happen
happily
happiness
happy
harbor
hard
hardly
hardship
hardware
hare
hark
harm
harness
harp
harvest
has
hasn't
haste (n)
hasty
hat
hatch
hatchet
hate
haul
have
haven't
having
hawk
hay
hayfield
haystack
he
head
headache
heal
health (y)
heap
hear (ing)
heard
heart
heat (er)
heaven
heavy
he'd
heel
height
held
hell
he'll
helmet

help (er)
helpful
hem
hen
henhouse
her (s)
herd
here
here's
hero
herself
he's
hey
hickory
hid
hidden
hide
high
highway
hill
hillside
hilltop
hilly
him
himself
hind
hint
hip
hire
his
hiss
history
hit
hitch
hive
ho
hoe
hog
hold (er)
hole
holiday
hollow
holy
home
homely
homesick
honest
honey
honeybee
honeymoon
honk
honor
hood
hoof
hook
hoop
hop
hope (ful)
hopeless
horn
horse
horseback
horseshoe
hose
hospital
host
hot
hotel
hound
hour
house
housetop
housewife
housework
how
however
howl
hug
huge
hum
humble
hump
hundred
hung
hunger
hungry
hunk
hunt (er)
hurrah
hurried
hurry
hurt
husband
hush
hut
hymn
I
ice
icy
I'd
ideal
if
ill
I'll
I'm
important
impossible
improve
in
inch (es)
income
indeed
Indian
indoors
ink
inn
insect
inside
instant
instead
insult
intend
interested
interesting
into
invite
iron
is
island

isn't
it
its
it's
itself
I've
ivory
ivy
jacket
jacks
jail
jam
January
jar
jaw
jay
jelly
jellyfish
jerk
jig
job
jockey
join
joke
joking
jolly
journey
joy (ful)
joyous
judge
jug
juice
juicy
July
jump
June
junior
junk
just
keen
keep
kept
kettle
key
kick
kid
kill (ed)
kind (ly)
kindness
king
kingdom
kiss
kitchen
kite
kitten
kitty
knee
kneel
knew
knife
knit
knives
knob
knock
knot
know
known
lace
lad
ladder
ladies
lady
laid
lake
lamb
lame
lamp
land
lane
language
lantern
lap
lard
large
lash
lass
last
late
laugh
laundry
law
lawn
lawyer
lay
lazy
lead
leader
leaf
leak
lean
leap
learn (ed)
least
leather
leave (ing)
led
left
leg
lemon
lemonade
lend
length
less
lesson
let
let's
letter
letting
lettuce
level
liberty
library
lice
lick
lid
lie
life
lift
light (ness)

lightning
like
likely
liking
lily
limb
lime
limp
line
linen
lion
lip
list
listen
lit
little
live (s)
lively
liver
living
lizard
load
loaf
loan
loaves
lock
locomotive
log
lone
lonely
lonesome
long
look
lookout
loop
loose
lord
lose (r)
loss
lost
lot
loud
love
lovely
lover
low
luck (y)
lumber
lump
lunch
lying
ma
machine
machinery
mad
made
magazine
magic
maid
mail
mailbox
mailman
major
make
making
male
mama
mamma
man
manager
mane
manger
many
map
maple
marble
march (M)
mare
mark
market
marriage
married
marry
mask
mast
master
mat
match
matter
mattress
may (M)
maybe
mayor
maypole
me
meadow
meal
mean (s)
meant
measure
meat
medicine
meet (ing)
melt
member
men
mend
meow
merry
mess
message
met
metal
mew
mice
middle
midnight
might (y)
mile
milk
milkman
mill
miller
million
mind
mine
miner
mint
minute

mirror
mischief
miss (M)
misspell
mistake
misty
mitt
mitten
mix
moment
Monday
money
monkey
manth
moo
moon
moonlight
moose
mop
more
morning
morrow
mom
most (ly)
mother
motor
mount
mountain
mouse
mouth
move
movie
movies
moving
mow
Mr. (s)
much
mud
muddy
mug
mule
multiply
murder
music
must
my
myself
nail
name
nap
napkin
narrow
nasty
naughty
navy
near
nearby
nearly
neat
neck
necktie
need
needle
needn't
Negro
neighbor
neither
neither
nerve
nest
net
never
nevermore
new
news
newspaper
next
nibble
nice
nickel
night
nightgown
nine
nineteen
ninety
no
nobody
nod
noise
noisy
none
noon
nor
north (ern)
nose
not
note
nothing
notice
November
now
nowhere
number
nurse
nut
oak
oar
oatmeal
oats
abey
ocean
o'clock
October
odd
of
off
offer
office
officer
often
oh
oil
old
old-fashioned
on
once
one
onion
only

onward
open
or
orange
orchard
order
ore
organ
other
otherwise
ouch
ought
our (s)
ourselves
out
outdoors
outfit
outlaw
outline
outside
outward
oven
over
overalls
overcoat
overeat
overhead
overhear
overnight
overturn
owe
owing
owl
own (er)
ox
pa
pace
pack
package
pad
page
paid
pail
pain (ful)
paint (er)
painting
pair
pal
palace
pale
pan
pancake
pane
pansy
pants
papa
paper
parade
pardon
parent
park
part (ly)
partner
party
pass
passenger
past
paste
pasture
pat
patch
path
patter
pave
pavement
paw
pay
payment
pea (s)
peace (ful)
peach (es)
peak
peanut
pear
pearl
peck
peek
peel
peep
peg
pen
pencil
penny
people
pepper
peppermint
perfume
perhaps
person
pet
phone
piano
pick
pickle
picnic
picture
pie
piece
pig
pigeon
piggy
pile
pill
pillow
pin
pine
pineapple
pink
pint
pipe
pistol
pit
pitch
pitcher
pity
place
plain
plan
plane

plant
plate
platform
platter
play (er)
playground
playhouse
playmate
plaything
pleasant
please
pleasure
plenty
plow
plug
plum
pocket
pocketbook
poem
point
poison
poke
pole
police
policeman
polish
polite
pond
ponies
pony
pool
poor
pop
popcorn
popped
porch
pork
possible
post
postage
postman
pot
potato (es)
pound
pour
powder
power (ful)
praise
pray
prayer
pr-pare
present
pretty
price
prick
prince
princess
print
prison
prize
promise
proper
protect
proud
prove
prune
public
puddle
puff
pull
pump
pumpkin
punch
punish
pup
pupil
puppy
pure
purple
purse
push
puss
pussy
pussycat
put
putting
puzzle
quack
quart
quarter
queen
queer
question
quick (ly)
quiet
quilt
quit
quite
rabbit
race
rack
radio
radish
rag
rail
railroad
railway
rain (y)
rainbow
raise
raisin
rake
ram
ran
ranch
rang
rap
rapidly
rat
rate
rather
rattle
raw
ray
reach
read
reader
reading
ready

real
really
reap
rear
reason
rebuild
receive
recess
record
red
redbird
redbreast
refuse
reindeer
rejoice
remain
remember
remind
remove
rent
repair
repay
repeat
report
rest
return
review
reward
rib
ribbon
rice
rich
rid
riddle
ride (r)
riding
right
rim
ring
rip
ripe
rise
rising
river
road
roadside
roar
roast
rob
robber
robe
robin
rock (y)
rocket
rode
roll
roller
roof
room
rooster
root
rope
rose
rosebud
rot
rotten
rough
round
route
row
rowboat
royal
rub
rubbed
rubber
rubbish
rug
rule (r)
rumble
run
rung
runner
running
rush
rust (y)
ruy
sack
sad
saddle
sadness
safe
safety
said
sail
sailboat
sailor
saint
salad
sale
salt
same
sand (y)
sandwich
sang
sank
sap
sash
sat
satin
satisfactory
Saturday
sausage
savage
save
savings
saw
say
scab
scales
scare
scarf
school
schoolboy
schoolhouse
schoolmaster
schoolroom
scorch
score
scrap

scrape
scratch
scream
screen
screw
scrub
sea
seal
seam
search
season
seat
second
secret
see (ing)
seed
seek
seem
seen
seesaw
select
self
selfish
sell
send
sense
sent
sent
sentence
separate
September
servant
serve
service
set
setting
settle
settlement
seven
seventeen
seventeenth
seventh
seventy
several
sew
shade
shadow
shady
shake (r)
shaking
shall
shame
shan't
shape
share
sharp
shave
she
she'd
she'll
she's
shear (s)
shed
sheep
sheet
shelf
shell
shepherd
shine
shining
shiny
ship
shirt
shock
shoe
shoemaker
shone
shook
shoot
shop
shopping
shore
short
shot
should
shoulder
shouldn't
shout
shovel
show
shower
shut
shy
sick (ness)
side
sidewalk
sideways
sigh
sight
sign
silence
silent
silk
sill
silly
silver
simple
sin
since
sing
singer
single
sink
sip
sir
sis
sissy
sister
sit
sitting
six
sixteen
sixth
size
skate
skater
ski
skin
skip

skirt
sky
slam
slap
slate
slave
sled
sleep (y)
sleeve
sleigh
slept
slice
slid
slide
sling
slip
slipped
slipper
slippery
slit
slow (ly)
sly
smack
small
smart
smell
smile
smoke
smooth
snail
snake
snap
snapping
sneeze
snow (y)
snowball
snowflake
snuff
snug
so
soak
soap
sob
socks
sod
soda
sofa
soft
soil
sold
soldier
sole
some
somebody
somehow
someone
something
sometime (s)
somewhere
son
song
soon
sore
sorrow
sorry
sort
soul
sound
soup
sour
south (ern)
space
spade
spank
sparrow
speak (er)
spear
speech
speed
spell (ing)
spend
spent
spider
spike
spill
spin
spinach
spirit
spit
splash
spoil
spoke
spook
spoon
sport
spot
spread
spring
springtime
sprinkle
square
squash
squeak
squeeze
squirrel
stable
stack
stage
stale
stall
stamp
stand
star
stare
start
starve
state
station
stay
steak
steal
steam
steamboat
steamer
steel
steep
steeple
steer
stem

step
stepping
stick (y)
stiff
still (ness)
sting
stir
stitch
stock
stocking
stole
stone
stood
stool
stoop
stop
stopped
stopping
store
stories
stork
storm (y)
story
stove
straight
strange (r)
strap
straw
strawberry
stream
street
stretch
string
strip
stripes
strong
stuck
study
stuff
stump
stung
subject
such
suck
sudden
suffer
sugar
suit
sum
summer
sun
Sunday
sunflower
sung
sunk
sunlight
sunny
sunrise
sunset
sunshine
supper
suppose
sure (ly)
surface
surprise
shallow
swam
swamp
swan
swat
swear
sweat
sweater
sweep
sweet (ness)
sweetheart
swell
swept
swift
swim
swimming
swing
switch
sword
swore
table
tablecloth
tablespoon
tablet
tack
tag
tail
tailor
take (n)
taking
tale
talk (er)
tall
tame
tan
tank
tap
tape
tar
tardy
task
taste
taught
tax
tea
teach (er)
team
tear
tease
teaspoon
teeth
telephone
tell
temper
ten
tennis
tent
term
terrible
test
than
thank (s)
thankful
Thanksgiving

that
that's
the
theater
thee
their
them
then
there
these
they
they'd
they'll
they're
they've
thick
thief
thimble
thin
thing
think
third
thirsty
thirteen
thirty
this
tho
thorn
those
though
thought
thousand
thread
three
threw
throat
throne
through
throw (n)
thumb
thunder
Thursday
thy
tick
ticket
tickle
tie
tiger
tight
till
time
tin
tinkle
tiny
tip
tiptoe
tire
tired
'tis
title
to
toad
toadstool
toast
tobacco
today
toe
together
toilet
told
tomato
tomorrow
ton
tone
tongue
tonight
too
took
tool
toot
tooth
toothbrush
toothpick
top
tore
torn
toss
touch
tow
toward (s)
towel
tower
town
toy
trace
track
trade
train
tramp
trap
tray
treasure
treat
tree
trick
tricycle
tried
trim
trip
trolley
trouble
truck
true
truly
trunk
trust
truth
try
tub
Tuesday
tug
tulip
tumble
tune
tunnel
turkey
turn
turtle
twelve

twenty
twice
twig
twin
two
ugly
umbrella
uncle
under
understand
underwear
undress
unfair
unfinished
unfold
unfriendly
unhappy
unhurt
uniform
United States
unkind
unknown
unless
unpleasant
until
unwilling
up
upon
upper
upset
upside
upstairs
uptown
upward
us
use (d)
useful
valentine
valley
valuable
value
vase
vegetable
velvet
very
vessel
victory
view
village
vine
violet
visit
visitor
voice
vote
way
wagon
want
wait
wake (n)
walk
wall
walnut
want
war
warm
warn
was
wash (er)
washtub
wasn't
waste
watch
watchman
water
watermelon
waterproof
wave
wax
way
wayside
we
weak (ness)
weaken
wealth
weapon
wear
weary
weather
weave
web
we'd
wedding
Wednesday
wee
weed
week
weep
weigh
welcome
well
we'll
went
were
we're
west (ern)
wet
we've
whale
what
what's
wheat
wheel
when
whenever
where
which
while
whip
whipped
whirl
whisky
whisper
whistle
white
who
who'd
whole
who'll

whom
who's
whose
why
wicked
wide
wife
wiggle
wild
wildcat
will
willing
willow
win
wind (y)
windmill
window
wine
wing
wink
winner
winter
wipe
wire
wise
wish
wit
witch
with
without
woke
wolf
woman
women
won
wonder
wonderful
won't
wood (en)
woodpecker
woods
wool
wollen
word
wore
work (er)
workman
world
worm
worn
worry
worse
worst
worth
would
wouldn't
wound
wove
wrap
wrapped
wreck
wren
wring
write
writing
written
wrong
wrote
wrung
yard
yarn
year
yell
yellow
yes
yesterday
yet
yolk
yonder
you
you'd
you'll
young
youngster
your (s)
you're
yourself
yourselves
youth
you've

"Quoted direct from Helping Children Read Mathematics.

APPENDIX H

LIST OF MATHEMATICS WORDS FAMILIAR TO 7TH-8TH GRADE STUDENTS*

List of Mathematics Words Familiar to

7th-8th Grade Students*

The plural formed by adding "s" or "es" to any word on the list is considered familiar.

add
addition
angle
area
arithmetic
average
base
base-ten
billion
borrowing
carrying
center
chart
check
circle
clockwise
closed
column
common
compare
comparing
compass
count
counting
cube
decimal
definition
diagram
diameter
difference
digit
direction
distance
divide
dividend
division
divisible
divisor
dot
double
eight
empty set
equal
equation
estimate
even
factor
figure
five
foot
four
fraction
fractional
graph
greater
group
half
height
horizontal
hour
improper
index
length
less
line
long division
lower
mathematical
mathematics
measure
measurement
million
mixed
multiple
multiplication
multiplier
multiply
nine
not
number
numerator
odd
one
opposite
pair
parallel
place
plane
plus
point
prime
problem

product
proper
quart
rectangle
remainder
right
Roman
round
rounded
rounding
ruler
sample
second
segment
sentence
set
seven
short
side
sign
simplest
six
solve
solving
square
straight
subtract
subtraction
sum
symbol
system
table
take away
ten
tens
term
testing
tests
three
total
triangle
true
two
units
unlike
value
vertical
weight
whole
width
word
yard
year
zero

*Quoted direct from Helping Children Read Mathematics.

APPENDIX I

LIST OF MATHEMATICS WORDS NOT FAMILIAR TO 7TH-8TH GRADE STUDENTS*

List of Mathematics Words Not Familiar to 7th-8th Grade Students*

Any word whose base word is on this list counts as Not Familiar. "Algebraic" is counted as Not Familiar since "algebra" is on this list.

If a word is on this list with an asterisk and as part of a particular phrase, only count the word as Not Familiar when it is used in the context given. "Find the square root" has two unfamiliar words since "*square*root" and "root" are on this list. "Draw a square" has no unfamiliar word since this meaning of square is not included in the phrases containing "square" that are on this list.

Any proper name used in a math phrase should be counted on the List of Unfamiliar Words. For example, "Babylonian numerals" counts as one unfamiliar mathematics word.

abacus
abscissa
absolute
accuracy
acute
addend
additive
adjacent
algebra
algorithm
alternate
altimeter
altitude
analogue
analysis
annexing
antilogarithm
antipodal
antipode
apex
apothem
approximate
arc
arcsine
argand
array
ascending
associative
assumption
asymptote
axiom
axis
Babylonian
balance
*bar*graph
*base*of parallelogram
binary*base*
betweenness
biased
biconditional
binary
binomial
bisect
bits
bolyai-lobachevskian
boundary
braces
*bridging*tens
*broken*curve
*broken*line
calculate
calibrated
cancel
capacity
*cardinal *number
caroid
caret
cartesian
*casting*out
celsius
central
centroid
characteristic
*Chinese *numerals
chord
circular

circumcenter
circumcircle
circumference
circumscribe
circumsphere
*clock*arithmetic
*closed*dot
*closed*figure
*closed*ray
*closed*region
*closed*sentence
closure
coefficient
cofunctions
collection
collinear
cologarithm
combinations
combine
commensurable
*common*divisor
*common*logarithm
commutative
comparable
comparison
complement
complete
complex
component
composite
compound
computation
compute
computer
concave
concentric
concurrent
condition
cone
conformal
congruent
conjecture
conjugate
conjun
conjugate
conjunction
consecutive
consequent
consistent
constant
construct
containment
continued
continuous
continuum
contrapositive
converse
convex
coordinates
coplanar
corollary
correspond
cosine
coterminal
conterexample
cross
cryptogram
cubed
cubit
cumulative
curve
cycle
cycloid
cyclometer
cylinder
data
decade
decagon
*decimal*notation
repeating*decimal*
decision
decoding
decrease
deduction
degree
*denominate*numbers
demoninator
dense
denumerable
dependent
depress
derivative
descending
determinants
deviation
diagonal
set*diagram*
dihedral
dimension
*direct
*proportion
*direct
*variation
*directed*angle
*directed
*distance
*directed
*numbers
*directed
*segment
*direction*of
a ray
directrix
discrete
discriminant
disjoint
disjunction
distribution
distributive
divisibility
dodecagon
domain
*dot*product
double angle
doubling
*draft*angle
duodecimal
eccentricity
edge
*eg
*Egyptian
*numerals

element
elevation
ellipse
empirical
enclose
end
endpoint
*English
*system
ensemble
enumerate
*equal*ratios
equality
equate
equator
equiangular
*equichord*point
equidistant
equilateral
equivalence
equivalent
error
estimating
*Euclidean*geometry
evaluate
event
exceed
exclusive
expanded
expansion
exponent
expression
exterior
external
extract
extraneous
extrapolation
extremes
face
factorial
factoring
factorization
false
family
fermat
field
finite
focal
focus
*foot*of a
perpendicular
form
formula
*fractional*equation
*fractional*form
*fractional*phrase
frame
frequency
frustrum
fulcrum
function
generate
geodesics
geometry
*geogonne*point
graphical
*great*circle
harmonic
hemisphere
heptagon
hexagon
*Hindu-Arabic numeral
histogram
hyperbola
hypotenuse
hypothesis
icosahedron
identify
*if-the*statements
imaginary
implication
incenter
incommensurable
inconsistent
increase
independent
indeterminant
indirect
induction
inequality
infinite
informal
initial
inner
inscribe
integer
integral
intercept
interior
internal
interpolate
intersect
interval
inverse
irrational
isometry
isomorphism
isosceles
join
joint
lateral
latitude
lattice
law
leading
leg
lemma
lever
like
limit
linear
literal
locus
logarithm
logic
*magic*square
magnitude
major
mantissa

map
matrix
maximum
*Mayan*digits
mean
*measure*of
   angle
*measure*of
   central
   tendency
median
member
mental
meridian
meter
method
metric
micro
midparallel
midplane
midpoint
minimum
minor
minuend
minus
*mixed
   *expression
mode
modular
modules
monomial
more
motion
multinomial
multiplicand
mutually
natural
necessary
negation
negative
*nine-point*circle
non-adjacent
nonagon
non-euclidean
non-linear
non-metric
non-negative
non-periodic
non-simple
nonsingular
non-terminating
norm
notation
null
numeration
numerical
oblique
obtuse
octagon
one-to-one
open
operation
*opposite*operations
*opposite*rays
order
ordinal
origin
orthocenter
orthogo
orthogonal
oval
*pair*of angles
parabola
paradox
*parallel*planes
*parallel*rays
parallelogram
parallelopiped
parameter
partial
path
pattern
pentagon
percentage
percentile
perfect
perimeter
period
*permissible
   *replacements
permutation
perpendicular
pi
pictograph
*picture*graph
placeholder
planar
*plane*angles
*plane*figure
X-Y*plane*
plot
*plumb*lines
polar
polygon
polyhedron
polynomial
position
positive
*possible*error
postulate
power
precise
*prime*factor
principal
principle
prism
probability
*product*set
profile
progression
projection
proof
property
proportion
proposition
protractor
puzzle
pyramid
pythagorean

quadrant
quadratic
quadrilateral
quantity
quartile
quaternion
*quinary*point
quotient
*radical
*segment
radian
radical
radicand
radii
radius
range
rate
ratio
*rational
*number
ray
*real*number
reasoning
reciprocal
*rectangular
*coordinate
system
*reduction
*formula
reference
reflexive
region
regroup
regular
relation
relative
*remote
*interior of
a triangle
renaming
*repetition
*principal
replacement
*representative
*number
result
resultant
*reuleaux*triangles
reverse
rhomboid
rhombus
*riemannian*geometry
*right*cylinder
*right*member
root
rotate
row
rule
*sacceri*quadrilateral
*sample*point
*sample*space
sampling
scale
scalene
scatter
*scientific*notation
secant
section
sector
segment
semicircle
separation
sequence
series
*set*builder notation
*set*of points
*shortest*path
*sieve*of Eratosthenes
*sign*of a number
radical*sign*
*signed*numbers
*significant*digit
similar
simple
simplex
simplify
*simson*line
*simultaneous*equations
sine
*sinusoidal*waves
*skew*line
*slant*height
*slide*rule
slope
*smooth*curve
solid
solution
space
span
sphere
*square*measure
*square*of a number
*square*root
standard
if-then*statements*
statistics
*straight*angle
*straight*edge
structure
subscript
subset
substitute
*subtend*of angle
subtrahend
sub unit
vector*sum*
summation
supplementary
surd
surface
sweep
symmetry
synthetic
coordinate*system*
tangent
computer*tape*
*target*set
tautology
technique

tendency
*term*of a polynomial
*terminal*side of angle
*terminating *decimal
*ternary *operations
tetrahedron
theorem
theory
topology
*transcendental *number
transform
transitive
translations
transposition
transversal
trapezoid
*triangle*of pascal
trichotomy
trihedral
trinomial
triple
tripod
trisection
*truth*set
*twin*primes
*two-point *equation
*two-point *form
unary
undefined
undoing
unequal
union
unique
*unit*circle
*unit*fraction
*unit*of measurement
*unit*of time
*unit*vector
*unit*of weight
*unit*of length
unknown
unlimited
valid
*value*of a function
*value*of a trigonometric ratio
variable
variance
variation
varies
vector
velocity
*Venn*diagram
*verbal*problems
vertex
vertical
vinculum
vinogradov
volume
wave
*well-defined *set
*zero*of a function

*Quoted direct from Helping Children Read Mathematics.

APPENDIX J

LIST OF MATHEMATICS SYMBOLS KNOWN

BY 7TH-8TH GRADE STUDENTS*

List of Mathematics Symbols Known

by 7th-8th Grade Students*

	as in
+	2 + 7
	8 3
x	2 x 3
$	$3.42
	6 - 3
%	3%
¢	2¢
any digit	any letter

*Quoted direct from Helping Children Read Mathematics.

APPENDIX K

MATHEMATICS LESSONS

MATHEMATICS LESSONS

	Page
Level A	
Lesson 1	112
Lesson 2	116
Lesson 3	119
Level B	
Lesson 1	122
Lesson 2	127
Lesson 3	130
Level C	
Lesson 1	132
Lesson 2	135
Lesson 3	138

LEVEL A

Lesson 1

Sets

The idea of a collection is very important in mathematics. In this lesson you will begin to see how this idea is used. Objects which belong to a set are said to be "members" or "elements" of the set. We can choose any object we wish and decide that they will make up a set. In our study we usually deal with sets whose names are numbers.

Suppose you want to talk about a set. You will need some way of making clear exactly what set you are talking about. One way would be to describe the set. You could describe one set by saying, "The set of students who have birthdays in May". You could describe another set by saying, "The set of numbers used in counting from 1 through 10". A description of a set must be so clear that the listener or reader can tell exactly which things belong to the set.

Think about the set whose members are Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, and Saturday. You can describe this as "The set of days of the week". Another way to describe this set would be to list all of the members. These two ways to describe sets will be explained below, but in both, we use capital letters as short names for sets and we place either the listing or the description within braces.

The method of listing the members of the set is called the "enumeration" method. Using the above set, let W be the set of days of the week. This is the way to describe this set by the enumeration method: W = {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}.

If you were to tell about the members of the set rather than list the members, you would be using a method called set builder notation.

W = {x x is a day of the week}

You read this "W is the set of all x that satisfy the condition that x is a day of the week". The straight line is read "that satisfy the condition" or more often "such that".

Let's look at another example of these two ways to tell about sets. Describe the set A of numbers 1,2,3,4.

Enumeration method (List)

A = {1,2,3,4}

Set Builder Notation (Describe)

A = {x x is one of the first four counting numbers}

or A = {x x < 4 and x is a counting number}

If we want to talk about one member of set A, we have a symbol to do this. $\in$ means "is an element or member of" and $\notin$ means "is not an element or member of". In set A, $1 \in A$ but $5 \notin A$.

When you are listing the members of a set, different order does not cause sets to be different. Also, you do not

have to list members but once. For example, the following sets are all equal: {1,1,2,3,3} and {2,3,1} and {3,2,1} and {1,2,3}.

The empty set

Count the number of green bears in this room. If there are none, then the set of green bears in this room is "the empty set". A set which has no members is called the empty set. The symbol for the empty set is $\emptyset$ or { }. Another name for the empty set is the null set. Note that C = {0} is a set having one member, the whole number 0. The set is not empty because it has one member in it. The empty set has no members. $\{0\} \neq \emptyset$.

Finite and Infinite sets

Look at set A again: A = {1,2,3,4}. Set A has four members. It is a "finite" set. It is possible to count the members of set A. It probably would take quite a long time to count all the members in a set which has 95,432 members. But, given enough time, they can be counted and described by a counting number. Hence, such a set is a finite set.

Let us start naming the counting numbers in order: 1,2, 3,4,5,6, Since we cannot complete the naming of the counting numbers, this is an infinite set. It is impossible to list all members of an infinite set. To show that there are more elements in an infinite set, we list a few, then write three dots. By looking at the pattern which is true

for the listed members, we can tell which other members belong to the set.

Remember that three dots can also be used to show a missing part in a finite set. The set B of counting numbers from 1 to 90 can be written as B = {1,2,3,4,...,90}.

Our two ways to describe sets works just as well for infinite sets. Using the set of counting numbers, look at the two following examples:

Enumeration method:

C = {1,2,3,4,...}

Set builder notation:

C = {x | x is a counting number}

You take the set of odd numbers and describe them by both methods:

Enumeration method Set builder notation

LEVEL A

Lesson 2

Set Relation

Subsets

Look at the following two sets. All of the members of set A also belong to set B.

$A = \{1,3,5\}$ and $B = \{1,2,3,4,5,6\}$

We call set A a "subset" of set B. Our definition says that if every member of set A is also a member of set B, then set A is a subset of set B. We use the symbol $\subset$ to show this. From this definition, you can see that every set is also a subset of itself. Study the following example:

$F = \{2,3,5\}$ and $G = \{10,5,6,3,2\}$

$F \subset G$ because every member of set F is also in set G.

$C = \{2,4,10\}$ and $D = \{2,3,10\}$

$C \subset D$ because every member of C is also in D.

$X = \{1,5,10\}$ and $Y = \{5,10,15\}$

It is not true that $X \subset Y$ because not every member of set X is also a member of set Y. We write: $X \not\subset Y$ (read: X is not a subset of Y). This means that at least one member of set X is not a member of set Y.

Proper Subset

The set A is a proper subset of set B if every member of set A is a member of set B, but not every member of set B

is a member of set A. In the sets F = {2,3,5} and G = {10,5,6,3,2}, F is a subset of G; but even more, F is a proper subset of G. This is shown by the symbol $\subset$. A proper subset has one more condition than a subset; the subset is not equal to the set. $F \subseteq G$ but $F \neq G$.

Equal sets

We say that two sets are "equal" when they have the same members. If R = {0,2,4} and S = {0, 3+1, 1+1} then R = S because 0 = 0, 1+1 = 2, and 3+1 = 4. Thus each set has the same members, the numbers 0,2,4. So, R and S are really two names for the same set.

Also, for any two sets R and S, if $R \subseteq S$ and $S \subseteq R$, then R = S.

For each pair of sets C and D tell whether or not C is a subset of D. Tell if it is also a proper subset.

(a) C = {the set of the thirteen original states}
D = {the set of all present states}

(b) C = {1,2,3,4} and D = {2,3}

(c) C = {2,4,6} and D = {2,4,6}

(d) C = {1,2} and D = {3,4,5}

When two sets do not have any members that are the same, they are said to be "disjoint". In example (d), sets C and D are disjoint.

Universal set

In talking about sets we will have in mind some fixed class of things to which we are limited, and the sets we use will be sets of members from this fixed class. The fixed class will be called the universal set. The universal set is not the same for all problems. With sets containing the numbers 1,2,3,4 the universal set could be the set of all counting numbers or the set of whole numbers. In other problems the universal set may be the set of all people, the set of all students, and so on. The symbol for the universal set is U.

LEVEL A

Lesson 3

Set Operation

Union

It is possible to have operations with sets just as it is with numbers. Look at the pairs of sets: C = {2,3,4} and T = {5,6,7}. We can form a new set, W, by putting together sets C and T. W = {2,3,4,5,6,7}. This operation of putting together is called "union". The symbol for union is U which looks like the letter "u".

Look:

Given V = {4,6,8} and H = {2,4,6}, remembering not to repeat any members of sets, the union of V U H = {2,4,6,8}.

Find the answer to the following:

(a) If R = {1,3,5} and T = {4,6,8,12} give R U T.

(b) If G = {the set of all girl students in your school}

B = {the set of all boy students in your school}

Give G U B.

(c) If D = {2,3,4} and E = {3,4,5,6,7} Give D U E.

Another way to show the union of two sets is with the use of Venn diagrams. Given sets A and B, the union can be shown by shading all of the two sets.

A U B

A U B

A U B

Intersection

We can also perform another operation with sets. If we take the set which has only those members which belong to both sets, we have the intersection of the two sets. The symbol used for intersection is

Look at these sets:

A = {2,3,5,7} and B = {5,7,1,13} then A ∩ B = {5,7}. Remember that two sets that have none of the same members are called disjoint sets. The intersection of two disjoint sets is the empty set.

The intersection of two sets can also be shown by using Venn diagrams. Given sets A and B, the shaded part shows the intersection of the two sets.

A ∩ B

A ∩ B

A ∩ B = ∅

Give the union and intersection for the following:

(a) If set S = {3,7,11,15} and set T = {11,15,19,23}

then S U T = and S ∩ T =

(b) If set L = 1,2 and set M = 3,4

then L M = and L ∩ M =

(c) Shade the following pictures to show A U B or A ∩ B.

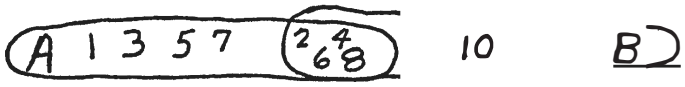
A U B

A B

A B

Another way to use the diagrams is shown below:

A = {1,2,3,4,5,6,7,8} and B = {2,4,6,8,10}



A ∩ B = {2,3,6,8}

A U B = {1,2,3,4,5,6,7,8,10}

LEVEL B

Lesson 1

Sets

Introduction

The use of sets to clarify the language and concepts of mathematics has increased in recent years through the introduction of "modern mathematics" in our school. Sets provide a basic for bringing together old and new concepts in mathematics and also privide a convenient language for explaining new mathematical concepts. In order to pursue the study of mathematics today, a student must become familiar with the language and symbolism of sets.

Sets

A "set" is a collection of objects. Other words, such as collection, class, group, or aggregate are also used in nonmathematical situations. For purposes of consistency, the word set is always used in mathematics. In order to better understand the use of sets, let us recall some nonmathematical uses. For example, we refer to a set of golf clubs, a chess set, the set of books in the library, etc. The student should be able to give other examples of sets that occur in everyday life.

The object of sets are called "elements" of the sets. We use lowercase letters to represent the elements of a set.

For example, the set A which has as its elements b,c,d, and e, but no other elements, can be written as A = {b,c,d,e}. The symbol used to show that an element "belongs to" a set is $\in$. In our example, b $\in$ A, c $\in$ A, d $\in$ A, and e $\in$ A. To indicate that a particular element "does not belong" to a set, we use the notation $\notin$. For instance, f $\notin$ A and g $\notin$ A.

We call this method of representing the set in the example "enumeration". For example, we can use enumeration to write the set B of all integers from 1 to 7 as B = {1,2,3,4,5,6,7}.

Example:

Use enumeration to describe the set M of the letters in the word Mississippi.

Solution: Although the word contains letters that are repeated, we only show each distinct letter once, so that M = {m,i,s,p}.

If a set is so defined that it does not have any elements, we call that set a null set (or empty set). The symbol for the null set is $\emptyset$ or { }. It is important to realize that 0 and $\emptyset$ do not mean the same thing. That is, $\emptyset$ is not equal to 0 or {0}, since {0} is a set with the one element 0, while $\emptyset$ (or { }) is the set with no elements; 0 represents the number of elements in $\emptyset$, although 0 is not an element of $\emptyset$.

Example:

Find the set P of all Presidents of the United States that died before their thirty-fifth birthday.

Solution: Since the requirements to be president of the United States include being at least thirty-five years old, this is an empty set; that is $P = \emptyset$.

A set is characterized by its elements. The order in which the elements appear is of no significance. The set A that we have discussed previously could also be enumerated as:

$A = \{e,c,b,d\} = \{d,e,c,b\}$ or any other possible order.

There are two common methods frequently used to describe sets. One is the enumeration method. The other is a set description method that employs the identifying properties of the elements of the set. For example, the set B of all positive integers between 1 and 100 could be described as follows: Let some letter, say x, represent an element of the set; then describe the set B and "the set of all x such that x is a positive integer between 1 and 100". That is,

$B = \{x \mid x \text{ is a positive integer between 1 and } 100\}$.

This set description, set builder notation, takes the form $A = \{x \mid x \text{ has property } p\}$, which is read "A is the set of all elements x such that x has the property p".

Example:

Use set builder notation to describe each of the

following sets:

(a) M, the set of all male citizens of the U.S.

(b) B, the set of all women who landed on the moon.

Solution: (a) $M = \{x \mid x \text{ is a male citizen of the U.S.}\}$

(b) Since no women have landed on the moon, B has no elements; that is $B = \emptyset$.

Finite and Infinite sets

Consider the following three sets: A is the set of boys in a mathematics class; B is the set of positive natural numbers from 1 to 100, and C is the set of months in a year. Sets A, B, and C can be enumerated, since every element of each set can be named explicitly. Such sets are examples of "finite" sets. In general, a set M is said to be a "finite" set if the elements in M can be exhaustively counted, in the usual way, using natural numbers and if the counting process eventually terminates, resulting in a specific number "n" (which is then said to be the number of elements in the set M). A set which is neither finite nor empty is called an infinite set. For example, the set D of all positive numbers is an infinite since enumeration is impossible in this case. It should be noted, however, that infinite sets whose elements form a general pattern can be described by enumeration. For example, the set D of all positive natural numbers can be enumerated as follows: $D = \{1,2,3,4,...\}$ where the three dots mean the same as "etc "

By contract, the set B of all positive natural numbers from 1 to 100 can be described as:

$$B = \{1,2,3,...,100\}$$

where the dots are used to represent the elements of B between 3 and 100.

LEVEL B

Lesson 2

Set Relations

Suppose that S is the set of students in a class and G is the set of girls in the same class. Clearly, all the members of G are also members of S. We describe the relation between G and S by saying that G is a subset of S. Symbolically, we write $G \subset S$. This can be read as "G is a subset of S", or "G is contained in S". In general, a set A is a subset of a set B, written $A \subseteq B$, if every element of A is also an element of B. That is, $A \subset B$ and $x \in A$ implies $x \in B$. For example, the set $A = \{1,2,3\}$ is a subset of a set $B = \{1,2,3,5\}$. Since every element of the set A is also an element of the set B.

We shall also agree that the empty set, $\emptyset$, is a subset of every other set. This agreement implies that every element of $\emptyset$ must also be an element of any other set, say A. Since $\emptyset$ contains no elements, this agreement does not contradict our definition of subset, and we may write $\emptyset \subset A$, for any set A.

Consider the set $A = \{m,n,c,d\}$ and $B = \{n,m,d,c\}$. Then, $A \subset B$ and $B \subset A$. Why? This suggests a definition of equality of sets, for if $A \subset B$ and $B \subseteq A$, then A and B have the same elements and we write $A = B$. Notice that if $A = \{1,2\}$

and B = {1,2,3}, then A _ B, but B A. In this case we say A is a proper subset of B. In general, A is a proper subset of B, written A B (the horizontal bar is left off) if all members of A are in B and B has at least one member not in A. That is, A B, but A ≠ B, implies A B.

Examples: If A = {1,2,3,4}, B = {1,2,3}, and C = {1,3,2} then B ⊂ A, C ⊂ B, and B ⊂ C. More precisely, B ⊂ A, C ⊆ A, and B = C.

List all the possible subsets of 1,2,3.

Solution: ∅, {1} , {2} , {3} , {1,2} , {1,3} , {2,3} , and {1,2,3} are the subsets of {1,2,3}. Note that the subsets, with the exception of {1,2,3} are proper subsets.

One should notice that the set of girls in a class and the set of boys in the same class have no elements in common. Also, the set A = {1,3,2} has no elements in common with set B = {5,6,7}. Such sets are called disjoint sets.

When we form subsets, there must be some source from which the elements of these subsets are to be selected; that is, any discussion of sets must be limited to some particular set. For example, the set A = {a,b,c,d,e} has elements which are letters of the alphabet. We call this larger set from which we select the elements to form subsets a universal set of the universe. The universe (designated as U) represents the complete set or largest set for the particular discussion, and all other sets in that same discussion will be

made up of elements from U. The choice of U will depend on the particular problem under discussion. The universal set U will not necessarily be the same for all problems. For example, in one problem U may be the set of all natural numbers; in another, U may be the set of all fractions.

LEVEL B

Lesson 3

Set Operation

Consider a universal set U = {1,2,3,...,8} and subsets of U, A = {1,2,3,4,5} and B = {3,4,5,6,7}. How can sets A and B be combined to form other subsets of U? One way is to combine all the elements of A and B to form the set {1,2,3,4,5,6,7}. The operation suggested by this example is called set union. In general, the union of two sets, written as A U B, is the set of all elements in set A or in set B, or in both A and B. Symbolically, A U B = {x | x $\in$ A or x $\in$ B (or both)}.

Venn diagrams, named for the English logician John Venn (1834-1883), can be used to represent pictorially the relations between sets. For example, if we let a universal set U be represented by some closed region, arbitrary sets A and B can be represented as other closed regions within the closed region representing U.

The shaded region in the figure above represents A U B. This figure represents A U B for A $\subset$ B, B $\subset$ A, and A and B are not disjoint.

The set formed by including all the elements common to both A and B from the above example is the set {3,4,5}.

This set is called the intersection of A and B. In general the intersection of two sets, written as $A \cap B$, is the set of all elements common to both A and B. Symbolically, $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$.

In the figure below, $A \cap B$, for $A \nsubseteq B$, $B \nsubseteq A$ and A and B are not disjoint, is represented by the shaded region.

In summary, the union of two sets is a third set which contains all the elements of the two given sets. The intersection of two sets is a third set which contains only those elements common to both sets. If A and B are subsets of some universal set U, then $A \cup B$, and $A \cap B$ are also subsets of U. If A and B are disjoint sets, then $A \cap B = \emptyset$. Note that $A \cup B$ is never the empty set unless $A = \emptyset$ and $B = \emptyset$. Why?

Example:

Determine $A \cup B$ and $A \cap B$ if $A = \{2,3,4,5,7\}$ and $B = \{5,6,3,7,8\}$.

Solution: $A \cup B = \{2,3,4,5,6,7,8\}$; $A \cap B = \{3,5,6\}$.

Note: Each element of A is an element of $A \cup B$ and each element of B is an element of $A \cup B$. Also each element of $A \cap B$ is an element of A and an element of B.

LEVEL C

Lesson 1

Set Theory

The words "set". "collect", "family", "bunch", and "herd" are all familiar to us. These usually denote a number of objects about which we wish to think collectively. We speak, for example, of a herd of dairy cows, a set of bone china dinner plates, and the members of some man's family. Although, as these examples show, the English language is rich in descriptive words for this basic concept, we shall generally use the word "set". Hence, in the above illustration we could have spoken of the set of dairy cows rather than the herd of dairy cows.

In any logical investigation certain terms must remain undefined. Any attempt to define every term would eventually lead to circular definitions. Thus, in this lesson, the term set and the concept of membership in a set will be taken as primitive, undefined concepts. For our purposes it will be sufficient to think of a set as being determined by the objects of which it is composed. Moreover, given any object or any object of any set, then either the object is in the set, or the object is not in the set. Alternatively, if a set is described as the collection of objects that satisfy a certain property, then for any object exactly one of these

two statements mustbe true: (1) the object has this property or (2) the object does not have this property.

To illustrate, if P is the property that an object is the cube of a positive integer, then it is clear that any object must either satisfy it. There is no middle ground. Thus all those objects that satisfy P form a set, namely, the set of third powers of positive integers.

Before we consider some examples of sets, it is convenient to agree on terminology and notation. Capital letters will usually be used to denote sets, lowercase letters will usually represent the objects of sets. If "a" is an object in the set A, we shall also say that "a" is an element of A or that a is a "member" of A or that "a" belongs to A. This will be denoted $a \in A$. If "a" is not an element of A, we write $a \notin A$. Frequently, we shall use the shorthand $A = \{x \mid x \text{ has property } P\}$ which is read "A is the set of all objects x such that x has property P". This is referred to as set builder notation. At times, when it is possible to list the elements of A, or at least to list sufficiently many to make clear just what are the elements of A, we shall write $A = \{x,y,z,...\}$. This notation is referred to as the enumeration method or sometimes the roster method.

In the preceding paragraphs it was not specifically stated that the sets mentioned actually had elements. A set having no elements is called an empty set or null set or

void set, some examples of which are "the set of three-dollar bills", "the set of all hairless apes", and "the set of all integers x such that $x^2 = 5$". The empty set shall be denoted by the symbol Ø, a letter of the Swedish alphabet or by { }.

Finite and Infinite sets

In general a set M is said to be a "finite" set if the elements in M can be counted; that is, placed in one-to-one correspondence with the natural numbers and terminating with a specific number. A set which is neither finite nor empty is referred to as "infinite". Infinite, as well as finite may be denoted by either set builder notation or by the enumeration method.

LEVEL C

Lesson 2

Set Relation

From the previous lesson, it is apparent that all the elements of one set may also be elements of a second set. For example, each element of the set P of positive integers is also an element of the set Z of all integers. Thus, if A and B are sets and each element of B is also an element of A, we say that B is a subset of A (B is included in A, B is contained in A, A contains B) and we write $B \subset C$ or $A \supset B$.

Some sets are defined by a complete listing of their elements. This listing, however, need not be unique. For example, let $A = \{1,2,1,1,3,4,2\}$, $B = \{1,2,3,4\}$, and $C = \{1,3,4,2\}$. Then, using the criterion for equality of sets discussed above, we see that $A = B = C$. For, even though 1 appears three times in the listing for A, $1 \in A$ and $1 \in B$. Similarly, $1 \in B$, $3 \in B$, and $4 \in B$, and thus $A = B$. Conversely, $B = A$ and so $A = B$. Similar arguments establish that $B = C$ and $A = C$. Hence, can you see that the number of times that an element appears in the listing is immaterial, as is the order of the listing?

If E is an empty set and F is any set whatever, we shall agree that $E \subseteq F$ This convention is not an unreasonable one, for E, having no elements, has no element that

does not belong to F. As a consequence of this agreement we can prove that there is only one empty set for, if G and H are empty sets, we have $G \subseteq H$ and $H \subseteq G$, which implies that $G = H$. Henceforth, we shall refer to the empty set. Do you see why?

If $B \subset A$ but $B \neq A$, we call B a proper subset of A and we write $B \subset A$. Clearly, $B \subset A$ means that: (1) if $x \in B$, then $x \in A$; and (2) there exists $y \in A$ such that $y \notin B$.

Example:

Let $A = \{1,2,3,4,5,6,7\}$ and $B = \{2,4,6\}$. Then $B \subset A$ and moreover, $B \subset A$.

In the event that B has an element x such that $x \in A$, then B is not a subset of A. This is indicated by $B \not\subseteq A$. Thus, $B = A$ if and only if $B \subset A$ and $A \subseteq B$.

One should be aware that if all elements, "a" which belong to set A are not contained in set B, sets A and B are said to be disjoint.

In most of our applications of set theory, the sets under consideration will be subsets of a fixed set called a universal set or a universe.

For example, let P_2 be the set of all squares of integers, P_3 the set of all cubes, and so forth. That is, for $n > 2$, let $P_n = \{x^n \mid x \text{ is an integer}\}$. Each set in our discussion is a subset of the set Z of all integers, which in this discussion may be taken as the universe. Moreover,

any union or intersection that we might form would again be a subset of Z.

Now suppose I is a universal set and A is a subset of I. Frequently, we may be interested in those elements in I that are not in A. For example, if I is the set of real numbers and if A is the set of rational numbers, then the set of elements of I that are not in A is precisely the set of all irrational numbers.

LEVEL C

Lesson 3

Set Operation

Given two sets we can combine them to form new sets. There is nothing sacred or particular about this number two; we can carry out the same procedure for any number of sets, finite or infinite. We do so for two first, because it illustrates the general construction but is not obscured by the additional notational difficulties.

The "union" of the two sets A and B, written A U B, is the set $\{x \mid x \in A \text{ or } x \in B\}$.

A word about the use of "or". In ordinary English when we say that something is "one or the other" we imply that it is not both. The mathematical "or" is quite different, at least when we are referring to set theory. For when we say that x is in A or x in in B, we mean x is in at least one of A or B and may be in both.

Let us consider a few examples of the union of two sets. For any set A, A U A = A; in fact, whenever B is a subset of A, A U B = A. If A is the set {1,2,3} (i.e., the set whose elements are 1,2,3) and if B is the set {4,5,1} then A U B = {1,2,3,4,5}. If A is the set of all blonde-haired people and if B is the set of all people who smoke, then do you follow that A B consists of all the people who either have

blonde-hair or that smoke or both? Pictorially with the use of Venn diagrams, we can illustrate the union of the two sets A and B by:

Here, A is the figure on the left, B that on the right, and A ∪ B is the shaded part.

The "intersection" of the two sets A and B, written A B, is the set $\{x \mid x \in A \text{ and } x \in B\}$.

The intersection of A and B is thus the set of all elements which are both in A and in B. In analogy with the example used to illustrate the union of two sets, let us see what the intersections are in those very examples. For any set A, $A \cap A = A$; in fact, if B is any subset of A, then $A \cap B = B$. If A is the set {1,2,3} and B the set {4,5,1} then $A \cap B = 1$. If A is the set of all blonde-haired people and if B is the set of all people that smoke, then $A \cap B$ is the set of all blonde-haired people who smoke. Pictorially we can illustrate the intersection of the two sets A and B by:

Here A is the figure on the left, B that on the right, while their intersection is the shaded section.

Two sets are said to be "disjoint" if their intersection is empty, that is, is the null set. For instance, if

A is the set of positive integers and B the set of negative integers, then A and B are disjoint. Note, however, that if C is the set of nonnegative integers and if D is the set of nonpositive integers, then they are not disjoint for their intersection consists of the integer 0, so the set is not empty.

APPENDIX L

PREDICTED CLOZE SCORES FOR LESSONS A, B, AND C

The following tables give the results of the application of Kane's formula II to the three levels of lessons. Each level of lesson A, B, and C was divided into as many 400-token sets as possible. Since the formula is intended for use with 400 tokens and no provisions are made for fewer than 400, the last few tokens in Lesson 3 of lesson groups A, B, and C were not included in the count. Groups A, and C each had four sets of 400 tokens. Group B had five sets of 400 tokens. Ranges of true cloze scores were determined from Kane's instructions as shown in Appendix C.

Level A Lesson

(Easy)

Predictor Variables

		A	B	$C_1 + C_2 = C$	D
Set of 400 Tokens	1	21	29	8 + 6 = 14	0
		20	43	11 + 7 = 18	0
	3	11	56	2 + 5 = 7	
		22	46	5 + 4 = 9	0
	TOTAL	74	174	48	0
	Mean	18.5	43.5	12	0

Predicted cloze score =

$$-.15(18.5)+ .10(43.5)- .42(12)- .17(0)+35.52$$

$$=-2.775 + 4.35 - 5.04 - 0 + 35.52$$

$$= 32.055$$

$$= 32.1$$

Level B Lesson

(Medium Difficulty)

		Predictor Variable			
		A	B	$C_1 + C_2 = C$	D
Set of 400 Tokens	1	52	30	7 + 5 = 12	0
	2	37	54	10 + 4 = 14	0
	3	56	31	12 + 7 = 19	1
	4	37	57	7 + 6 = 13	1
	5	52	37	23 + 8 = 31	0
	TOTAL	234	209	89	2
	Mean	46.8	41.8	17.8	.4

Predicted cloze score =
-.15(45.8)+ .10(41.8)-.42(17.8)-.17 .4)+35.52
=-7.02 + 4.18 - 7.476 - .68 + 35.52
= 24.524
=24.5

Level C Lessons

(Difficult)

Predictor Variable

		A	B	$C_1 + C_2 = C$	D
Set of 400 Tokesn	1	77	6	12 + 7 = 19	0
	2	67	10	18 + 8 = 26	0
	3	51	56	9 + 14 = 23	2
	4	43	22	13 + 8 = 21	1
TOTAL		238	94	89	3
Mean		59.5	23.5	22.25	.75

Predicted cloze scores =

$-.15(59.5) + .10(23.4) - .42(22.25) - .17(.75) + 35.52$

$= 8.925 + 2.35 - 9.345 - .1275 + 35.52$

$= 18.325$

$= 18.3$

Range of True Cloze Scores

Level	A(32.1)	B(24.5)	C(18.3)
90%	±2.3	±2.7	±5.3
	34.4 - 29.3	26.5 - 22.5	21.8 - 14.8

The readability levels of lessons A and B are significantly different at the .90 level of confidence. Lesssons B and C have readability levels that are significantly different only at the .75 level of confidence.

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