Chapter 5: Continuous Random Variables

5.1: Continuous Probability Functions

- Probability density function (PDF) a probability distribution for a continuous random variable, sometimes thought of as the curve that would be attained from smoothing a histogram obtained through infinite sampling

- Properties of continuous probability distributions

  - Notated as $f(x)$, but $f(x) \neq P(X = x)$

  - Rather, $P(X = a) = 0$ for all a

  - $P(a < X < b)$ is the area under the curve from a to b

  - $f(x) \geq 0$ for all values of x

  - $f(x) = 0$ for all $x \notin S$

  - The total area under the curve equals one

- Key idea: area = probability

Chapter 6: The Normal Distribution

6.1: The Standard Normal Distribution

The standard normal distribution is a normal distribution of **standardized values called z-scores**.

A z-score is measured in units of the standard deviation.

For example, if the mean of a normal distribution is five and the standard deviation is two, the value 11 is three standard deviations above (or to the right of) the mean.

The mean, μ, for the standard normal distribution is 0, and the standard deviation, σ, is 1.

The transformation $z = \dfrac{x-\mu}{\sigma}$ produces the distribution $Z \sim N(0,1)$. The value x in the given equation comes from a normal distribution with mean μ and standard deviation σ.

Properties of the Standard Normal Distribution

- The graph for the Standard Normal Distribution is symmetric and bell-shaped.
- The mean for the Standard Normal Distribution is 0 with a standard deviation of 1.
- The normal curve is symmetric about the mean, μ, such that half of the data is to the left and half falls to the right of the mean, μ.
- The mean, median, and mode are all centered in the middle of the Standard Normal Distribution.
- The total area under the curve is 1.

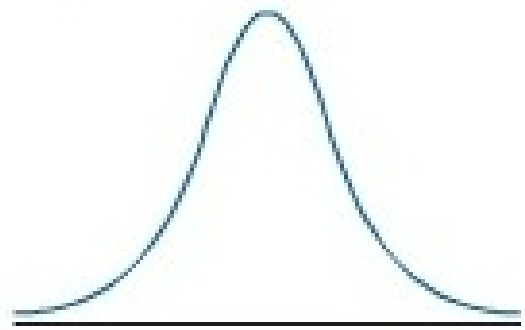
Properties of ANY Normal Distribution

- The graph for a Normal Distribution is symmetric and bell-shaped.
- The normal curve is symmetric about the mean, μ, such that half of the data is to the left and half falls to the right of the mean, μ.
- The mean, median, and mode are all centered in the middle of a Normal Distribution.
- The total area under the curve is 1.

6.2: Using the Normal Distribution

Normal Curve - many continuous random variables have relative frequency histograms with a shape similar to Figure **??**. They are said to have the shape of a normal curve.

Figure 1:



A continuous random variable is **normally distributed**, or has a **normal probability distribution**, if the relative frequency histogram of the random variable has the shape of a normal curve.

For symmetric distributions with a single peak, such as the normal distribution, the mean = median = mode. Because of this, the mean, μ, is the high point of the graph of the distributions.

Finding Probabilities When Given z Scores:

- The z-table will be used to find probabilities of z scores. The area associated with a given z score on the table refers to the cumulative probability for z (area to the left). That is, it refers to $P(Z \leq z)$.
  - *z score*: Distance along the horizontal scale of the standard normal distribution; refer to the leftmost column and top row of the z-table.
  - *Area*: Region under the curve; refer to the values in the body of the z-table.

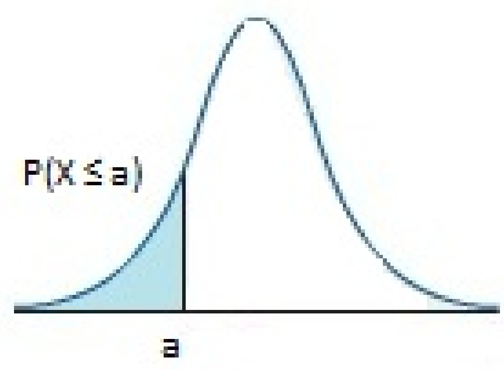
Notation for the Probability of a Standard Normal Random Variable

$P(a < Z < b)$ represents the probability that a standard normal random variable is between a and b.

$P(Z > a)$ represents the probability that a standard normal random variable is greater than a.

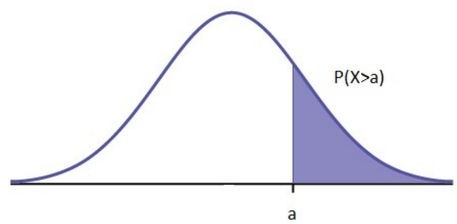
$P(Z < a)$ represents the probability that a standard normal random variable is less than a.

Cumulative Probability: the probability that a random variable is less than or equal to a specified value.

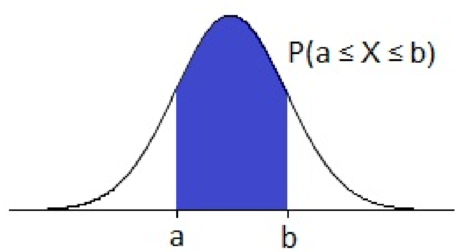


Useful Identities:

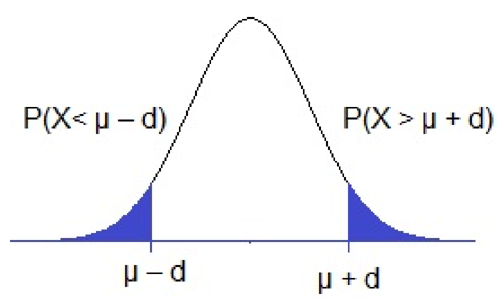
1. $P(X > a) = 1 - P(X \leq a)$



2. $P(a < X < b) = P(X \leq b) - P(X \leq a)$



3. $P(X > \mu + d) = P(X < \mu - d)$.



4. $P(X = a) = 0$ for all a.

Example 1.

(a) $P(Z > 2.45)$

(b) $P(Z < 0.39)$

(c) $P(-0.25 < Z < 0.25)$

(d) $P(Z < C) = 0.60$, where C is the cutoff value.

(e) $P(Z > C) = 0.28$, where C is the cutoff value.

(f) Find the z-scores that separate the middle 80% of the data.

Find the Area under the Standard Normal Curve

Example 2. Assume that the heights of college women have a normal distribution with mean $\mu = 65$ inches and standard deviation $\sigma = 2.7$ inches. What is the probability that a randomly selected college woman is 62 inches or shorter?

Area to the Right of Z

But, how do we find the area to the RIGHT of z? We know the area under the entire curve is equal to 1, thus

$$\text{(Area under the normal curve to the right of } z \text{)} = 1 - \text{(Area to the left of } z \text{)}$$

Example 3. Now find the probability a randomly selected college woman is taller than 70 inches.

Area Between Two z-Scores

Example 4. Find the probability that a randomly selected college woman will be between 60 and 68 inches.

Step 1 Draw a picture.

Step 2 Find z-scores for both x-values.

Step 3 Find the area to the left of both z-scores from Step 2.

Step 4 Subtract the areas.

Finding Percentiles

Step 1 Draw a normal curve and shade the area corresponding to the proportion, probability, or percentile given.

Step 2 Use the z-table to find the z-score that corresponds to the shaded area.

Step 3 Obtain the normal value from the formula $x = \mu + z\sigma$.

Example 5. Suppose that the blood pressure of men aged 18 to 29 years old have a normal distribution with mean $\mu = 120$ and standard deviation $\sigma = 10$. What value of blood pressure is the 75th percentile for this population?

Example 6. The heights of a pediatrician's 200 three-year-old females are approximately normally distributed with mean 38.72 inches and standard deviation 3.17 inches. Find the height of a 3-year-old female at the 20th percentile. That is, find the height of a 3-year-old female that separates the bottom 20% from the top 80%.

Example 7: General Electric manufactures a decorative Crystal Clear 60-watt light bulb that it advertises will last 1,500 hours. Suppose that the lifetimes of the light bulbs are approximately normally distributed, with a mean of 1,550 hours and a standard deviation of 57 hours.

(a) What probability of the light bulbs will last less than the advertised time?

(b) What probability of the light bulbs will last more than 1,650 hours?

(c) What is the probability that a randomly selected GE Crystal Clear 60-watt light bulb will last between 1,625 and 1,725 hours?

(d) What is the probability that a randomly selected GE Crystal Clear 60-watt light bulb will last exactly 1400 hours?

(e) How long will a light bulb last if it is in the top 15% of all working light bulbs?

(f) What is the length of time a light bulb lasts if it is at the 8th percentile of all working light bulbs?

Example 8. A pediatrician obtains the heights of her 200 3-year-old female patients. The heights are approximately normally distributed, with mean 38.72 inches and standard deviation 3.17 inches. The pediatrician wishes to determine the heights that separate the middle 98% of the distribution from the bottom 1% and the top 1%. In other words, find the 1st and 99th percentiles.

